Optimal Debt Maturity and Firm Investment*

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Abstract

We introduce long-term debt and a maturity choice into a dynamic model of production, firm financing, and costly default. Long-term debt saves roll-over costs but increases future leverage and default rates because of a commitment problem. The model generates rich distributions of maturity choices, leverage ratios, and credit spreads across firms. It explains why larger and older firms borrow at longer maturities, have higher leverage, and pay lower credit spreads. Firms' maturity choice matters for policy: A financial reform which increases investment and output in a standard model of short-term debt can have the opposite effect in a model with short-term debt and long-term debt.

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1. Introduction

This paper starts out from a simple observation. Empirically, most firm debt is long-term. About 67% of the average U.S. corporation's total stock of debt does not mature within the next year. This fact is missing from most macroeconomic models. The standard assumption is that all firm debt is short-term, i.e. all debt issued in period t fully matures in period $t+1$. In this paper, we introduce long-term debt and a maturity choice into a dynamic model of production, firm financing, and costly default. We find that the model replicates important cross-sectional facts on firm financing and debt maturity. Moreover, we show that firms' maturity choice matters: Policy recommendations based on a model of endogenous debt maturity can be the opposite of those generated by a standard model of short-term debt.

In this paper, we study a model of heterogeneous firms which finance productive capital with equity, short-term debt, and long-term debt. Each period, firms choose investment, leverage, and debt maturity. Long-term debt saves roll-over costs but increases future leverage and default rates because of a commitment problem: Outstanding long-term debt distorts firms' incentives to issue additional debt (*debt dilution*) and to invest (*debt overhang*). These effects are particularly large if a firm's default risk is high.

We then take this model to the data. Combining balance sheet information on US publicly listed firms with data on credit ratings and bond spreads, we document large and systematic heterogeneity in firms' investment, financing, and maturity choices. Smaller and younger firms have lower shares of long-term debt. Their leverage ratios are low and credit spreads on their debt are high. As firms grow in size and age, their share of long-term debt increases. Larger and older firms have higher leverage ratios and pay lower credit spreads.

Our model replicates these patterns. The key mechanism is that the costs of higher debt maturity (i.e. the negative effects of debt dilution and debt overhang) are particularly large for firms with high default risk. Because larger and older firms are more profitable, their default risk is low. This reduces the cost of borrowing at long maturities and allows them to increase leverage and pay lower credit spreads compared to smaller and younger firms.

We then use the model to study two counterfactuals. A first experiment quantifies the costs of debt dilution and debt overhang in distorting firms' investment, borrowing, and maturity choices. We find that debt dilution and debt overhang increase leverage and the default rate, while reducing debt maturity and investment. In a second experiment, we show that it is important to take firms' maturity choice into account in model-based policy evaluations. A financial reform which lowers default costs increases investment, output, and consumption in a standard model of short-term debt. We show that the same reform can have the opposite effect in a model of endogenous debt maturity because it gives rise to an increase in debt dilution and debt overhang.

Our paper provides a quantitative analysis of endogenous debt maturity in a dynamic model of production, firm financing, and costly default. It contributes to a large literature that studies the role of financial frictions in shaping firm dynamics. Existing work typically assumes that all firm debt is short-term (e.g. Cooley and Quadrini, 2001;

Hennessy and Whited, 2005; Gilchrist, Sim, and Zakrajšek, 2014; Khan, Senga, and Thomas, 2016; Ottonello and Winberry, 2018; Arellano, Bai, and Kehoe, 2019). From an empirical point of view, the disregard of long-term debt is problematic. At issuance, the average term to maturity is three to four years for bank loans, and more than eight years for corporate bonds (Adrian, Colla, and Shin, 2012).

Computational difficulties are the main reason why risky long-term debt is usually absent from dynamic macroeconomic models. Optimal firm behavior depends on the price of long-term debt, which itself depends on firm behavior, both today and in the future. A firm that cannot commit to future actions must take into account how today's choices will affect future firm behavior. In this paper, we compute the global solution to this fixed point problem. This allows us to study how firms optimally adjust their debt structure over time and how these choices shape the firm distribution.

Gomes, Jermann, and Schmid (2016) consider long-term debt but assume exogenous debt maturity. Our results suggest that models which take firms' maturity choice into account can contribute to our understanding of firm dynamics and the impact of policy reforms.[1]

There is a long tradition in corporate finance of modelling firms' maturity choice. In Leland and Toft (1996), Leland (1998), Diamond and He (2014), DeMarzo and He (2016), and Dangl and Zechner (2016), firms are not allowed to adjust the maturity structure of their debt over time. In Brunnermeier and Oehmke (2013) and He and Milbradt (2016), firms are allowed to dynamically adjust maturity but the total value of debt is fixed. We contribute to this theoretical literature by providing a quantitative analysis which allows for the dynamic adjustment of both the level and the maturity of debt.

In the sovereign debt literature, a substantial body of work studies the maturity structure of government debt (e.g. Debortoli, Nunes, and Yared, 2017; Faraglia, Marcet, Oikonomou, and Scott, 2018). The interaction of government debt maturity and default risk is studied by Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Hatchondo, Martinez, and Sosa-Padilla (2016), and Aguiar, Amador, Hopenhayn, and Werning (2019). While many insights from this literature carry over to models of firm financing, endogenous output and investment play an important additional role for firms' maturity choice.

In Section 2 we develop a dynamic model of production, leverage, and debt maturity. Section 3 evaluates the quantitative performance of the model. In Section 4 we use the model to conduct two counterfactual experiments. Concluding remarks follow.

[1] In Caggese and Perez (2016) and Paul (2018), the level of long-term debt is non-adjustable. Karabarbounis and Macnamara (2019) study a model of exogenous debt maturity. Crouzet (2017) highlights several challenges in modelling risky long-term debt and matching empirical maturity structures. Poeschl (2018) and Jungherr and Schott (2019) focus on the role of debt maturity over the business cycle.

2. Model

Firms produce output using capital and labor. Capital is financed through equity and debt. Debt has a tax advantage relative to equity but introduces the risk of costly default. Firms can choose between short-term debt and long-term debt. Long-term debt saves roll-over costs but increases future leverage and default risk because of a commitment problem.

The model economy consists of a mass of heterogeneous firms, a representative household, and a government. The household saves by buying equity and debt securities issued by firms. The government collects a corporate income tax and pays out the proceeds to the representative household as a lump-sum transfer. All agents take the wage w and the riskless rate r as given. Because there is no aggregate risk, factor prices are constant over time.

2.1. Firm setup

At time t a firm i uses capital k_{it} and labor l_{it} to produce output according to:

$$y_{it} = z_{it} \left(k_{it}^{\psi} l_{it}^{1-\psi} \right)^{\zeta}, \quad \text{with:} \quad \zeta, \psi \in (0,1), \tag{1}$$

where z_{it} is firm-specific productivity. Earnings before interest and taxes are

$$y_{it} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f, \tag{2}$$

where ε_{it} is a firm-specific capital quality shock, δ is the depreciation rate, and f is a fixed cost of operation.

There are two sources of uncertainty. Productivity z_{it} is persistent and is realized at the end of period $t-1$ with conditional probability distribution $\pi(z_{it}|z_{it-1})$. The capital quality shock ε_{it} is i.i.d. with mean zero and continuous probability distribution $\varphi(\varepsilon)$. It is realized after production has taken place in period t. An example for a negative capital quality shock is an unforeseen change in technology or consumer demand which reduces the value of existing firm-specific capital.

The firm can finance capital with equity, short-term debt, and long-term debt.

Definition: Short-term debt. A short-term bond issued at the end of period $t-1$ is a promise to pay one unit of the numéraire good in period t together with a fixed coupon c. The quantity of short-term bonds sold by firm i and due in period t is $\tilde{b}_{it}^{S}$.

Let p_{it-1}^{S} be the market price of short-term debt issued by firm i at the end of period $t-1$. If the firm sells a quantity $\tilde{b}_{it}^{S}$ of short-term bonds, it raises an amount $\tilde{b}_{it}^{S} p_{it-1}^{S}$ on the short-term bond market.

Definition: Long-term debt. A long-term bond issued at the end of period $t-1$ is a promise to pay a fixed coupon c in period t. In addition, the firm repays a fraction $\gamma \in (0,1)$ of the principal in period t. In period $t+1$, a fraction $1-\gamma$ of the bond remains outstanding. The firm pays a coupon $(1-\gamma)c$ and repays the fraction γ of the remaining principal. In this manner, payments decay geometrically over time. The maturity parameter γ controls the speed of decay. The quantity of long-term bonds chosen by the firm at the end of period $t-1$ is $\tilde{b}^L_{it}$.

This computationally tractable specification of long-term debt goes back to Leland (1994). Let b_{it-1} denote the stock of previously issued long-term bonds outstanding at the end of period $t-1$, and let p^L_{it-1} be the market price of long-term debt issued by firm i. If a firm increases its level of long-term debt to $\tilde{b}^L_{it}$ by selling additional bonds, it raises the amount $(\tilde{b}^L_{it} - b_{it-1})p^L_{it-1}$ on the long-term bond market. Short-term debt and long-term debt are of equal seniority.

Definition: Debt issuance cost. The firm pays a quadratic issuance cost whenever it sells new short-term or long-term bonds. Repurchasing outstanding long-term debt (by choosing $\tilde{b}^L_{it} < b_{it-1}$) is costless. Total debt issuance costs $H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1})$ are

$$H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1}) = \eta \left(\tilde{b}^S_{it} + \max\{\tilde{b}^L_{it} - b_{it-1}, 0\}\right)^2 . \tag{3}$$

The issuance cost makes short-term debt unattractive because it needs to be constantly rolled over. Long-term debt matures slowly over time and therefore allows maintaining a given stock of debt at a lower level of bond issuance per period.[2]

The firm finances its capital stock by injecting equity and by selling new short- and long-term bonds. Let q_{it-1} be the stock of assets in place, and let e_{it-1} denote net equity issuance at the end of period $t-1$, that is, the net cash flow from shareholders to the firm. A negative value of e_{it-1} indicates a net dividend payment from the firm to shareholders. Capital in period t is given by:

$$k_{it} = q_{it-1} + e_{it-1} + \tilde{b}^S_{it} p^S_{it-1} + (\tilde{b}^L_{it} - b_{it-1})p^L_{it-1} - H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1}) \tag{4}$$

Firm earnings are taxed at rate τ. Debt coupon payments are tax deductible. The stock of firm assets in period t after production and repayment of debt is

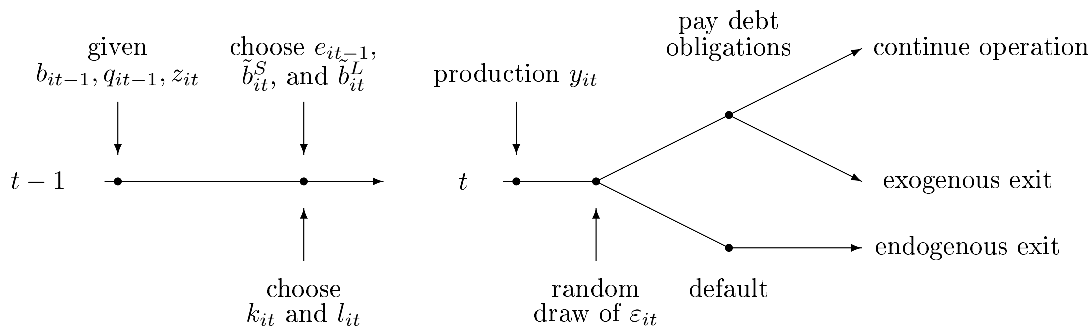
$$q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma \tilde{b}^L_{it} + (1-\tau)\left[y_{it} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f - c(\tilde{b}^S_{it} + \tilde{b}^L_{it})\right] . \tag{5}$$

The fact that coupon payments are tax deductible lowers total tax payments by the amount $\tau c(\tilde{b}^S_{it} + \tilde{b}^L_{it})$. This is the benefit of debt. The downside is that the firm cannot commit to repaying its debt.

Definition: Limited liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. Default is costly. Creditors recover only a fraction $1-\xi$ of firm assets.

[2] The same mechanism would also apply to a linear issuance cost instead of a quadratic one. Altınkılıç and Hansen (2000) provide empirical evidence that marginal debt issuance costs are increasing in debt issuance.

Figure 1: Timing



A defaulting firm exits the economy. In addition to this, there is exogenous exit. With probability κ, a non-defaulting firm exogenously leaves the economy. In this case, the exiting firm repurchases any outstanding stock of long-term debt at the market value $b_{it}p^L_{it}$. The remaining firm value $q_{it} - b_{it}p^L_{it}$ is paid out to shareholders.

The timing is summarized in Figure 1. At the end of period $t-1$, a firm has an amount b_{it-1} of long-term debt outstanding and assets q_{it-1}. Firm productivity z_{it} is realized. The firm chooses capital k_{it} and labor l_{it}. Capital is financed by issuing equity e_{it-1} and by selling short-term bonds $\tilde{b}^S_{it}$ and additional long-term bonds $\tilde{b}^L_{it} - b_{it-1}$. In period t, the firm produces output y_{it}. The idiosyncratic capital quality shock ε_{it} is realized and the firm decides whether to default. Exogenous exit occurs with probability κ. Continuing firms have an amount $b_{it} = (1-\gamma)\tilde{b}^L_{it}$ of long-term debt outstanding. Firm assets are q_{it}.

2.2. Firm problem

Firms maximize shareholder value, that is, the expected present value of net cash flows to shareholders. They discount cash flows at the risk-free rate r. Shareholder value of a firm which continues to operate at the end of period $t-1$ can be written as the sum of assets in place and a term which depends on firm behavior: $q_{it-1} + V_{t-1}(b_{it-1}, z_{it})$. Because there are no equity issuance costs, the amount of assets in place, q_{it-1}, has no influence on the optimal firm policy and the value $V_{t-1}(b_{it-1}, z_{it})$.

We describe the firm problem starting from the default decision at the end of period t. The idiosyncratic capital quality shock ε_{it} has been realized but future firm productivity z_{it+1} (and therefore $V_t(b_{it}, z_{it+1})$ and p^L_{it}) is still uncertain at this point. If the firm does not default, expected shareholder value is

$$(1-\kappa)\left[q_{it} + \mathbb{E}\, V_t\left(b_{it}, z_{it+1}\right)\right] + \kappa\left[q_{it} - b_{it}\,\mathbb{E}\, p^L_{it}\right], \tag{6}$$

where the expectation $\mathbb{E}$ is taken over future firm productivity z_{it+1} conditional on z_{it}. With probability $1-\kappa$, the firm continues to operate. Exogenous exit occurs with probability κ. Assets after production q_{it} are increasing in ε_{it}. Limited liability protects shareholders from large negative realizations of ε_{it}. There exists a unique threshold realization $\bar{\varepsilon}_{it}$ which sets expected shareholder value (6) to zero:

$$\bar{\varepsilon}_{it}: \quad q_{it} + (1-\kappa)\,\mathbb{E}\,V_t\,(b_{it}, z_{it+1}) - \kappa\, b_{it}\,\mathbb{E}\,p^L_{it} = 0 \tag{7}$$

If ε_{it} is smaller than $\bar{\varepsilon}_{it}$, full repayment would result in negative expected shareholder value, whereas default provides an outside option of zero. In that case, the firm optimally defaults on its liabilities. The threshold value $\bar{\varepsilon}_{it}$ depends on the firm's choice of debt and capital. By choosing a ratio of debt to capital at the end of period $t-1$, the firm controls the default threshold $\bar{\varepsilon}_{it}$ and thereby the probability of default.

At the end of period $t-1$, the firm chooses its scale of production and its preferred financing mix. The firm anticipates that shareholder value will be positive if ε_{it} is higher than the threshold value $\bar{\varepsilon}_{it}$ and zero otherwise. Given a stock of assets in place q_{it-1}, existing debt b_{it-1}, and productivity z_{it}, a firm solves:

$$\max_{\substack{k_{it}, l_{it}, e_{it-1} \geq \underline{e}, \\ \tilde{b}^S_{it}, \tilde{b}^L_{it}}} \quad -e_{it-1} + \frac{1}{1+r} \int_{\bar{\varepsilon}_{it}}^{\infty} \left[q_{it} + (1-\kappa)\,\mathbb{E}\,V_t\,(b_{it}, z_{it+1}) - \kappa\, b_{it}\,\mathbb{E}\,p^L_{it} \right] \varphi(\varepsilon) d\varepsilon \tag{8}$$

$$\begin{aligned}
\text{subject to:} \quad & q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma \tilde{b}^L_{it} + (1-\tau) \left[y_{it} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f - c(\tilde{b}^S_{it} + \tilde{b}^L_{it}) \right] \\
& y_{it} = z_{it} \left(k^{\psi}_{it} l^{1-\psi}_{it} \right)^{\zeta} \\
& \bar{\varepsilon}_{it}: \quad q_{it} + (1-\kappa)\,\mathbb{E}\,V_t\,(b_{it}, z_{it+1}) - \kappa\, b_{it}\,\mathbb{E}\,p^L_{it} = 0 \\
& k_{it} = q_{it-1} + e_{it-1} + \tilde{b}^S_{it} p^S_{it-1} + (\tilde{b}^L_{it} - b_{it-1}) p^L_{it-1} - H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it-1}) \\
& b_{it} = (1-\gamma) \tilde{b}^L_{it}
\end{aligned}$$

The firm's choice of e_{it-1} is bounded from below: $e_{it-1} \geq \underline{e}$, with $\underline{e} < 0$. This constitutes an upper limit for dividend payments.[3]

The optimal firm policy crucially depends on the two bond prices p^S_{it-1} and p^L_{it-1}. A high bond price implies a low credit spread which reduces the firm's cost of capital. We now derive the firm-specific bond prices from the creditors' optimization problem.

2.3. Creditors' problem

Creditors are perfectly competitive and break even on expectation. They buy firm bonds at the end of period $t-1$. If the firm does not default in period t, short-term creditors receive $(1+c)\tilde{b}^S_{it}$, and long-term creditors are paid $(\gamma + c)\tilde{b}^L_{it}$. In case of default, the total

[3] If the stock of existing debt b_{it-1} is sufficiently large, the firm may find it optimal to choose a corner solution and pay out the entire asset value of the firm as dividend: $e_{it-1} = -q_{it-1}$. In practice, it is illegal to pay dividends which substantially exceed firm earnings and deplete a firm's stock of capital. We choose the value of the constraint $\underline{e}$ such that it rules out this corner solution but is not binding in equilibrium. The exact value of $\underline{e}$ does not affect equilibrium variables.

value of the firm's assets is

$$\underline{q}_{it} \equiv k_{it} + (1-\tau)(y_{it} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f)\,. \tag{9}$$

At this point, creditors liquidate the firm's assets and receive $(1-\xi)\underline{q}_{it}$. Short- and long-term debt have equal seniority. The break-even price of short-term debt is

$$p^S_{it-1} = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon}_{it})](1+c) + \frac{(1-\xi)}{\tilde{b}^S_{it} + \tilde{b}^L_{it}} \int_{-\infty}^{\bar{\varepsilon}_{it}} \underline{q}_{it} \varphi(\varepsilon) d\varepsilon\right], \tag{10}$$

where $1-\Phi(\bar{\varepsilon}_{it})$ is the probability that $\varepsilon_{it} > \bar{\varepsilon}_{it}$. The price of short-term debt only depends on firm behavior at time t, in particular on the risk of default $\Phi(\bar{\varepsilon}_{it})$. In contrast, the price of long-term debt p^L_{it-1} also depends on the future market value of long-term debt $p^L_{it} = g_t(b_{it}, z_{it+1})$:

$$p^L_{it-1} = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon}_{it})]\left[\gamma + c + (1-\gamma)\,\mathbb{E}\, g_t(b_{it}, z_{it+1})\right] + \frac{(1-\xi)}{\tilde{b}^S_{it} + \tilde{b}^L_{it}} \int_{-\infty}^{\bar{\varepsilon}} \underline{q}_{it} \varphi(\varepsilon) d\varepsilon\right]. \tag{11}$$

If the firm does not default in period t, it repays a fraction γ of the outstanding debt plus the coupon c. A fraction $1-\gamma$ of the debt remains outstanding. Because the future price of long-term debt $p^L_{it} = g_t(b_{it}, z_{it+1})$ depends on future firm behavior, it is a function of the future state of the firm. The firm cannot directly control future firm behavior, but it can influence the future bond price through today's choice of long-term debt: $b_{it} = (1-\gamma)\tilde{b}^L_{it}$.

2.4. Equilibrium firm policy

In equilibrium, a firm maximizes shareholder value (8) subject to creditors' break-even conditions (10) and (11). Because we assume that the firm has no ability to commit to future actions, it must take its own future behavior as given and chooses today's policy as a best response. In other words, the firm plays a game against its future selves. As in Klein, Krusell, and Rios-Rull (2008), we restrict attention to the Markov perfect equilibrium, i.e. we consider policy rules which are functions of the payoff-relevant state variables. The time-consistent policy is a fixed point in which the future firm policy coincides with today's firm policy.

The value $V_{t-1}(b_{it-1}, z_{it})$ can be computed recursively. We define the sum of assets in place q_{it-1} and equity issuance e_{it-1} as a choice variable: $\tilde{e}_{it-1} \equiv q_{it-1} + e_{it-1}$. Time subscripts are dropped in the recursive formulation of the problem. Each period, the firm chooses a policy vector $\phi(b,z) = \{k, l, \tilde{e}, \tilde{b}^S, \tilde{b}^L\}$ which solves

$$V(b,z) = \max_{\phi(b,z)=\left\{\substack{k,l,\tilde{e}\geq\underline{\tilde{e}},\\ \tilde{b}^S,\tilde{b}^L}\right\}} -\tilde{e} + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty} \left[q + (1-\kappa)\,\mathbb{E}\,V(b',z') - \kappa\, b'\,\mathbb{E}\,g(b',z')\right]\varphi(\varepsilon)d\varepsilon \tag{12}$$

$$\begin{aligned}
\text{s.t.:}\quad & q = k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)\left[y + \varepsilon k - wl - \delta k - f - c(\tilde{b}^S + \tilde{b}^L)\right] \\
& y = z\left(k^{\psi} l^{1-\psi}\right)^{\zeta} \\
& \bar{\varepsilon}:\quad q + (1-\kappa)\,\mathbb{E}\,V(b',z') - \kappa\, b'\,\mathbb{E}\,g(b',z') = 0 \\
& k = \tilde{e} + \tilde{b}^S p^S + (\tilde{b}^L - b)p^L - H(\tilde{b}^S, \tilde{b}^L, b) \\
& b' = (1-\gamma)\tilde{b}^L \\
& p^S = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\ \varphi(\varepsilon)d\varepsilon\right] \\
& p^L = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})]\left[\gamma + c + (1-\gamma)\mathbb{E}\,g(b',z')\right] + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\ \varphi(\varepsilon)d\varepsilon\right]
\end{aligned}$$

Firm choices are heterogeneous because of differences in firm productivity z and firms' existing stock of debt b. In addition, firm outcomes differ because of the idiosyncratic capital quality shock ε. In the absence of equity issuance costs, past earnings do not affect the current optimal firm policy $\phi(b,z)$. The amount of assets in place q is not a state variable of the firm problem.

2.5. Firm entry & exit

Firms exit the economy endogenously because of default, and exogenously at rate κ. There is free entry. A new firm starts without existing debt and with an initial level of productivity $z = z^e$. New firms enter the economy as long as the value of entry $V(0, z^e)$ is positive. The mass of entrants is denoted as M.

2.6. Households

We close the model by introducing a representative household who owns all equity and debt claims issued by firms and receives all income in the economy. Government revenue from taxation is paid out to the household as a lump-sum transfer. The household works, consumes, and invests its savings in equity and debt.

Future utility is discounted at rate β. We assume additive-separable preferences over consumption C_t and labor L_t. Period utility is

$$\ln(C_t) - \frac{L_t^{1+\theta}}{1+\theta}. \tag{13}$$

2.7. General equilibrium

We study the stationary distribution of the economy. Let $\tilde{b}^L(b,z)$ and $\bar{\varepsilon}(b,z)$ denote the firm's choice of long-term debt and the default threshold as a function of its state. The law of motion for the firm distribution is

$$\mu'(b',z') = \int_0^\infty \int_0^\infty \mu(b,z)\,\pi(z'|z)\,(1-\kappa)\,\left[1-\Phi\left(\bar{\varepsilon}(b,z)\right)\right]\,\mathcal{I}(b',b,z)\,db\,dz \,+\, \mathcal{M}(b',z'), \tag{14}$$

where the indicator function $\mathcal{I}(b',b,z)=1$ if $b'=(1-\gamma)\,\tilde{b}^L(b,z)$. The function $\mathcal{M}(b',z')$ is equal to M at $b'=0$ and $z'=z^e$, and zero otherwise. A stationary distribution μ^* is a fixed point of (14).

Definition: Stationary equilibrium. A stationary equilibrium consists of *(i)* a policy vector $\phi(b,z)=\{k,l,\tilde{e},\tilde{b}^S,\tilde{b}^L\}$, a value function $V(b,z)$, and bond price functions p^S and p^L, *(ii)* a stationary distribution μ^* and a mass of entrants M^*, *(iii)* aggregate labor supply L^* and household consumption C^*, and *(iv)* a wage w^* and an interest rate r^*, such that:

1. $\phi(b,z)$, $V(b,z)$, p^S, and p^L solve the firm problem (12).
2. The free entry condition holds: $V(0,z^e)=0$.
3. The representative household chooses C^* and L^* optimally.
4. The labor market and the goods market clear.

Because there is no aggregate risk and any given firm has zero weight in the representative household's portfolio, all equity and debt claims are priced as if households were risk neutral. The return on the representative household's aggregate portfolio is certain and equal to the riskless rate $r^*=1/\beta-1$.

Optimal labor supply L^* is determined by the household's first order condition $w/C = L^{*\theta}$. Let $l(b,z)$ be a firm's labor demand. Labor market clearing implies that

$$L^* = \int_0^\infty \int_0^\infty l(b,z)\,\mu^*(b,z)\,db\,dz\,. \tag{15}$$

Goods market clearing implies that

$$\begin{aligned} Y \equiv \int_0^\infty \int_0^\infty \Big[y(b,z) - f - H\big(\tilde{b}^S(b,z),\tilde{b}^L(b,z),b\big) \\ - \,\xi \int_{-\infty}^{\bar{\varepsilon}(b,z)} \underline{q}\,\varphi(\varepsilon)\,d\varepsilon\Big]\,\mu^*(b,z)\,db\,dz \,=\, C\,+\,I\,, \end{aligned} \tag{16}$$

where C is household consumption and aggregate investment I is

$$I = \delta \int_0^\infty \int_0^\infty k(b,z)\,\mu^*(b,z)\,db\,dz\,. \tag{17}$$

2.8. Characterization

The firm problem (12) can be expressed in terms of only three choice variables: the scale of production k and the amounts of long-term debt $\tilde{b}^L$ and short-term debt $\tilde{b}^S$. Accordingly, the equilibrium behavior of firms is characterized by three first order conditions. For simplicity, we derive these optimality conditions assuming that long-term debt issuance is positive ($\tilde{b}^L - b > 0$), that there is no exogenous exit ($\kappa = 0$), and that the liquidation value of a firm is zero ($\xi = 1$). All derivations are deferred to Appendix A.

With $\xi = 1$, the short-term bond price only depends on the default threshold $\bar{\varepsilon}$:

$$p^S = \frac{1}{1+r}\left[1 - \Phi(\bar{\varepsilon})\right](1+c) \tag{18}$$

The long-term bond price is a function of $\bar{\varepsilon}$ and $b' = (1-\gamma)\tilde{b}^L$:

$$p^L = \frac{1}{1+r}\left[1 - \Phi(\bar{\varepsilon})\right]\left[\gamma + c + (1-\gamma)\,\mathbb{E}\,g\,(b', z')\right] \tag{19}$$

The default threshold $\bar{\varepsilon}$ depends on firm actions through (7). As shown in Appendix A, $\bar{\varepsilon}$ is a function of k, $\tilde{b}^S$, and $\tilde{b}^L$. By choosing its scale of production k, long-term debt $\tilde{b}^L$, and short-term debt $\tilde{b}^S$, the firm controls the default threshold $\bar{\varepsilon}$ and thereby the probability of default.

Capital The firm's first order condition with respect to capital k is:

$$-1 + \frac{\partial \bar{\varepsilon}}{\partial k}\left[\tilde{b}^S \frac{\partial p^S}{\partial \bar{\varepsilon}} + (\tilde{b}^L - b)\frac{\partial p^L}{\partial \bar{\varepsilon}} - \frac{1-\tau}{1+r}k[1 - \Phi(\bar{\varepsilon})]\right] + \frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}(\varepsilon - \bar{\varepsilon})\varphi(\varepsilon)d\varepsilon = 0 \tag{20}$$

For a given quantity of short-term and long-term bonds sold, increasing capital by one additional unit is a net injection of equity into the firm and has a cost of one. Increasing capital affects the firm's default threshold $\bar{\varepsilon}$:

$$\frac{\partial \bar{\varepsilon}}{\partial k} = -\frac{1 + (1-\tau)(\mathrm{MPK} + \bar{\varepsilon} - \delta)}{(1-\tau)k}, \tag{21}$$

where MPK is the firm's marginal product of capital. If increasing capital lowers the risk of default (i.e. $\partial\bar{\varepsilon}/\partial k < 0$), the benefits consist in higher bond market revenue from selling short-term ($\partial p^S/\partial\bar{\varepsilon}<0$) and long-term debt ($\partial p^L/\partial\bar{\varepsilon}<0$), and in higher dividend payments (in case default is avoided).

High values of existing debt b can decrease investment. If $\partial\bar{\varepsilon}/\partial k < 0$, the firm's optimal choice of k is falling in the stock of existing debt b. This is because a higher market price of long-term bonds benefits shareholders only to the extent that the firm sells new long-term bonds to creditors. The fact that lower default risk also raises the market value of existing debt is not internalized by the firm. This is the classic *debt* *overhang* effect described by Myers (1977).

Short-term debt The firm's first order condition with respect to $\tilde{b}^S$ is

$$\frac{1-\Phi(\bar{\varepsilon})}{1+r}\tau c+\frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^S}\left[\tilde{b}^S\frac{\partial p^S}{\partial\bar{\varepsilon}}+(\tilde{b}^L-b)\frac{\partial p^L}{\partial\bar{\varepsilon}}\right]-2\eta(\tilde{b}^S+\tilde{b}^L-b)=0 \tag{22}$$

Selling short-term debt benefits shareholders because with probability $1-\Phi(\bar{\varepsilon})$ default is avoided and total tax payments are reduced by the amount τc. The downside consists of an increase of the probability of default:

$$\frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^S}=\frac{1+(1-\tau)c}{(1-\tau)k}>0 \tag{23}$$

Higher default risk lowers the revenue from selling new debt on the bond market $\tilde{b}^S p^S+(\tilde{b}^L-b)p^L$ because it reduces equilibrium bond prices: $\partial p^S/\partial\bar{\varepsilon}<0$ and $\partial p^L/\partial\bar{\varepsilon}<0$. In addition, the firm incurs the marginal debt issuance cost.

Because $\partial\bar{\varepsilon}/\partial\tilde{b}^S>0$ and $\partial p^L/\partial\bar{\varepsilon}<0$, the firm's optimal choice of $\tilde{b}^S$ is increasing in the stock of existing debt b. As explained above, the firm does not internalize potential default costs which pertain to the holders of existing long-term debt. While the firm fully internalizes the tax benefits of additional debt, it only internalizes part of the associated costs. If b is high relative to $\tilde{b}^L$, long-term debt issuance $\tilde{b}^L-b$ is small. This reduces the part of expected default costs which is internalized by the firm through the bond market. This incentive to increase indebtedness at the expense of existing creditors is known as *debt dilution.*[4]

Long-term debt The firm's first order condition with respect to $\tilde{b}^L$ is:

$$\begin{aligned}\frac{1-\Phi(\bar{\varepsilon})}{1+r}\mathbb{E}&\left[\tau c+(1-\gamma)\left(g\left(b',z'\right)+(\tilde{b}^L-b)\frac{\partial g\left(b',z'\right)}{\partial\tilde{b}^L}\right)+\frac{\partial V\left(b',z'\right)}{\partial\tilde{b}^L}\right]\\&+\frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^L}\left[\tilde{b}^S\frac{\partial p^S}{\partial\bar{\varepsilon}}+(\tilde{b}^L-b)\frac{\partial p^L}{\partial\bar{\varepsilon}}\right]-2\eta\left(\tilde{b}^S+\tilde{b}^L-b\right)=0\end{aligned} \tag{24}$$

As for the case of short-term debt, high values of existing long-term debt b encourage further issuance of long-term debt because of *debt dilution.* Combining (24) with the optimality condition for short-term debt (22) yields a condition for firms' optimal maturity

[4] A remark on terminology: In corporate finance, the term *debt dilution* is sometimes used for the specific situation that an increased number of creditors needs to share a given liquidation value of a bankrupt firm. We use the term in a more general sense as the same mechanism is at work even if the liquidation value is zero or if existing debt is fully prioritized (as in Bizer and DeMarzo, 1992). In our usage of the term *debt dilution* we therefore follow the literature on sovereign debt (e.g. Hatchondo et al., 2016).

choice:

$$\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,\mathbb{E}\left[(1-\gamma)\,2\eta\,(\tilde{b}^{S'}+\tilde{b}^{L'}-b')+(\tilde{b}^{L}-b)(1-\gamma)\frac{\partial g\,(b',z')}{\partial\tilde{b}^{L}}\right]$$
$$-\left[\tilde{b}^{S}\frac{\partial p^{S}}{\partial\bar{\varepsilon}}+(\tilde{b}^{L}-b)\frac{\partial p^{L}}{\partial\bar{\varepsilon}}\right]\mathbb{E}\left[1-g(b',z')+2\eta\,(\tilde{b}^{S'}+\tilde{b}^{L'}-b')\right]\frac{1-\gamma}{(1-\tau)k}=0 \quad (25)$$

Issuing long-term debt instead of short-term debt has two benefits: (1.) With probability $1-\Phi(\bar{\varepsilon})$ default is avoided. In this case, the firm saves an amount $(1-\gamma)2\eta(\tilde{b}^{S'}+\tilde{b}^{L'}-b')$ of future issuance costs. (2.) The probability of default falls because of a higher future shareholder value. This raises the market value of newly issued debt, $\tilde{b}^{S}p^{S}+(\tilde{b}^{L}-b)p^{L}$. Future shareholder value increases for two reasons: (i) Because $g(b',z')<1$, a given stock of long-term debt tomorrow is less of a debt burden for the firm than the same amount of short-term debt. The reason is that the firm discounts the future at a rate higher than $1/(1+r)$ because of default risk. (ii) Future issuance costs are lowered.

The only downside of higher debt maturity is that a higher future outstanding stock of debt $b'=(1-\gamma)\tilde{b}^{L}$ affects future firm behavior and thereby reduces tomorrow's long-term bond price: $\partial g(b',z')/\partial\tilde{b}^{L}<0$. As explained above, the firm's future choice of capital can be falling in b' because of *debt overhang*. The firm's future choice of short-term and long-term debt is increasing in b' because of *debt dilution*. Both of these effects increase future default risk and lower the future price of long-term debt. Because creditors are forward-looking, this already reduces today's market price of long-term debt.

In the absence of default risk, a firm's bond price is independent of firm behavior. *Debt dilution* and *debt overhang* affect existing creditors through the effect of a firm's investment and borrowing decisions on the probability of default. Because *debt dilution* and *debt overhang* are particularly severe for firms with high default risk, the negative effect of high maturity on today's price of long-term debt is stronger for riskier firms. For this reason, riskier firms optimally choose a lower share of long-term debt. Firms with low default risk will rely more on long-term debt because their costs of *debt dilution* and *debt overhang* are low.

3. Quantitative analysis

The Markov perfect equilibrium in (12) can only be computed using numerical methods. In this section, we lay out our computational approach, describe the data sources used, discuss the calibration strategy, and present quantitative results.

3.1. Solution method

We find the global solution to the dynamic firm problem in (12) using value function iteration and interpolation. The key difficulty consists in finding the equilibrium price of long-term debt p^{L}. Optimal firm behavior depends on p^{L} which itself depends on current and future firm behavior. We solve this fixed point problem by computing the solution to a finite-horizon problem. Starting from a final date, we iterate backward until all prices and quantities have converged. We then use the first-period equilibrium allocation as the equilibrium of the infinite-horizon economy. This means that we iterate simultaneously on the value and the long-term bond price (as in Hatchondo and Martinez, 2009). The presence of the idiosyncratic i.i.d. capital quality shock ε with continuous probability distribution $\varphi(\varepsilon)$ facilitates the computation of p^L (*cf.* Chatterjee and Eyigungor, 2012).

To solve for general equilibrium we proceed as follows. First, the equilibrium wage rate is determined by free entry. The stationary distribution of firms μ^* is then computed using firms' equilibrium policies $\phi(b, z) = \{k, l, \tilde{e}, \tilde{b}^S, \tilde{b}^L\}$. The total mass of firms is pinned down by labor market clearing.

3.2. Data

Our main dataset combines balance sheet data from Compustat with data on corporate bond issues from the Mergent Fixed Income Securities Database (FISD) for the years 1984-2018 (see Appendix B for details).

From Compustat, we obtain quarterly firm-level information on investment, leverage, long-term debt shares, and credit ratings. Leverage is defined as total firm debt divided by total firm assets. The long-term debt share is the amount of debt with remaining term to maturity of more than one year divided by total firm debt. We restrict our firm sample to US non-financial firms and exclude firms in the public and utility sectors. Our final sample consists of 7,859 unique firms and 324,825 observations.

For data on corporate bond spreads, we use FISD data on US bond issues. The dataset contains bond-level information on yields, credit ratings, and maturity. As in the Compustat sample, we only consider bonds issued by US firms outside of the financial, public, or utility sector. Our final sample consists of 31,063 bond issues. Credit spreads are computed as bond yields at issuance minus the yield of a US treasury of identical maturity issued on the same day. Using the median credit spread across bond issues of a given credit rating class in a given quarter, we calculate time series of credit spreads broken down by rating class. We use this data to proxy a firm's credit spread in a given quarter by the median spread of the corresponding rating class.[5]

3.3. Calibration

The model period is one quarter. We set $\beta = 0.99$ which implies a quarterly rate of return on a riskless asset $r^* = 1.01\%$ and corresponds to an annual return of $(1+r^*)^4 - 1 = 4.1\%$. The debt coupon is $c = r^*$ which implies that the equilibrium price of a riskless short-term and long-term bond are both equal to one. The preference parameter θ is chosen to generate a steady state labor elasticity of 0.5 (Chetty, Guren, Manoli, and Weber, 2011). The production technology parameters ζ and ψ are taken from Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018). The quarterly depreciation rate δ is 2.5%.

[5] See Arellano et al. (2019) for a similar approach to proxy for firm-level credit spreads.

Table 1: Preassigned parameters

Parameter	Description	Value	Source
β		0.99	
c	debt coupon	$1/\beta - 1$	
θ	inverse labor elasticity	2	Chetty et al. (2011)
ζ	technology parameter	0.75	Bloom et al. (2018)
ψ	technology parameter	0.33	Bloom et al. (2018)
δ	depreciation rate	0.025	
γ	repayment rate long-term debt	0.05	Gomes et al. (2016)
τ	corporate income tax rate	0.4	Gomes et al. (2016)

We follow Gomes et al. (2016) in setting the repayment rate of long-term debt $\gamma = 0.05$ and the tax rate $\tau = 0.4$. This choice of γ implies a Macaulay duration of $(1 + r^*)/(\gamma + r^*) = 16.8$ quarters or 4.2 years. This is a conservative choice relative to the average duration of 6.5 years calculated by Gilchrist and Zakrajsek (2012) for a sample of US corporate bonds with remaining term to maturity above one year. These parameters are summarized in Table 1.

The probability distribution of the firm-specific capital quality shock ε is assumed to be Normal with zero mean and standard deviation σ_ε. Firm-level productivity z follows a productivity ladder with discrete support $\{Z_1, ..., Z_j, ..., Z_J\}$. Entrants start with the lowest productivity level $z^e = Z_1$. Incumbent firms with last period's productivity level $z = Z_j$ climb up the productivity ladder with probability $1 - \rho_z$:

$$z' = \begin{cases} Z_j & \text{with probability } \rho_z \\ Z_{\min\{j+1,J\}} & \text{with probability } 1 - \rho_z \end{cases} \tag{26}$$

Once a firm has reached the highest productivity level Z_J, it remains there until it defaults or exits the economy exogenously. The support of the natural logarithm of z is evenly spaced on the interval $\pm\,\sigma_z$.

This productivity process has two desirable features. First, it captures the positive skewness of empirical firm growth. Large negative firm growth is rare in the data.[6] Second, it facilitates the computation of the Markov perfect equilibrium. Negative productivity shocks decrease the value $V(b, z)$ while the existing stock of debt b remains unchanged. If a shock is sufficiently large, the incentive to pay out firm assets to shareholders at the expense of existing creditors causes the constraint $\tilde{e} \geq \underline{\tilde{e}}$ in (12) to bind for any value of $\underline{\tilde{e}}$. The productivity ladder described above does not feature large negative

[6] In our sample, the cross-sectional investment rate has a skewness of 4.6. The frequency of investment rate observations that exceed the sample mean by more than one standard deviation is 6.5%. In contrast, only 0.8% of investment rates are smaller than one standard deviation below the sample mean.

Table 2: Jointly identified parameters

Parameter	Value	Target	Data	Model
σ_ε	0.627	Average firm leverage	32.8%	32.6%
ξ	0.900	Average credit spread on long-term debt	2.5%	2.4%
η	0.004	Average share of long-term debt	66.9%	66.4%
ρ_z	0.965	Median of average net investment rate	0.7%	0.6%
σ_z	0.290	Median of s.d. of net investment rates	3.5%	3.4%
κ	0.017	Total exit rate (quarterly)	2.2%	2.3%
f	0.303	Unit mass of firms	-	1.00

Note: Leverage, the long-term debt share, and investment rates are from Compustat. Firm-level investment rates are net of the aggregate investment rate. Credit spreads are computed using data from Compustat and FISD. The exit rate is from Ottonello and Winberry (2018). See Appendix B for additional details.

jumps in $V(b, z)$ and thereby avoids this problem. The constraint $\tilde{e} \geq \underline{\tilde{e}}$ is not binding in equilibrium.[7]

The remaining seven parameters σ_ε, ξ, η, ρ_z, σ_z, κ, and f are jointly chosen to match the moments in Table 2. While the model is highly non-linear and all parameters are identified jointly, we provide some intuition for the identification of the model parameters. Average firm leverage depends on the standard deviation of the capital quality shock, σ_ε, because higher volatility induces firms to reduce leverage in order to contain the risk of default. The average credit spread is directly affected by the default cost ξ. The issuance cost parameter η is pinned down by the average share of long-term debt because higher issuance costs make short-term debt less attractive. The parameters ρ_z and σ_z determine the mean and the standard deviation of investment rates. The probability of exogenous exit κ affects the overall rate of exit (exogenous and endogenous through default) in the economy. Finally, the fixed cost of operation f is chosen to generate a unit mass of firms.

The model matches the data very well. The average leverage ratio is 32.6% and around two-thirds of firm debt is long-term. The average credit spread on long-term debt is 2.4%. The model generates a total quarterly exit rate of 2.3%. The quarterly default rate is 0.6% which lies between the value of 0.3% targeted by Gomes et al. (2016) and the business failure rate of 0.8% used in Bernanke, Gertler, and Gilchrist (1999).[8]

The average and the standard deviation of firm-level investment rates are close to the data as well.[9] The model also replicates the positive skewness of empirical investment

[7] An alternative would be to use a standard AR(1) firm productivity process. Because in this case the dividend payout constraint would bind for many firms, the particular form of this constraint would significantly affect model results.

[8] The average of Moody's expected quarterly default frequency across rated and unrated Compustat firms is 1.0% (Hovakimian, Kayhan, and Titman, 2011).

[9] To make the data comparable to our stationary model, we construct net investment rates by subtracting the aggregate rate of investment from firm-level investment rates.

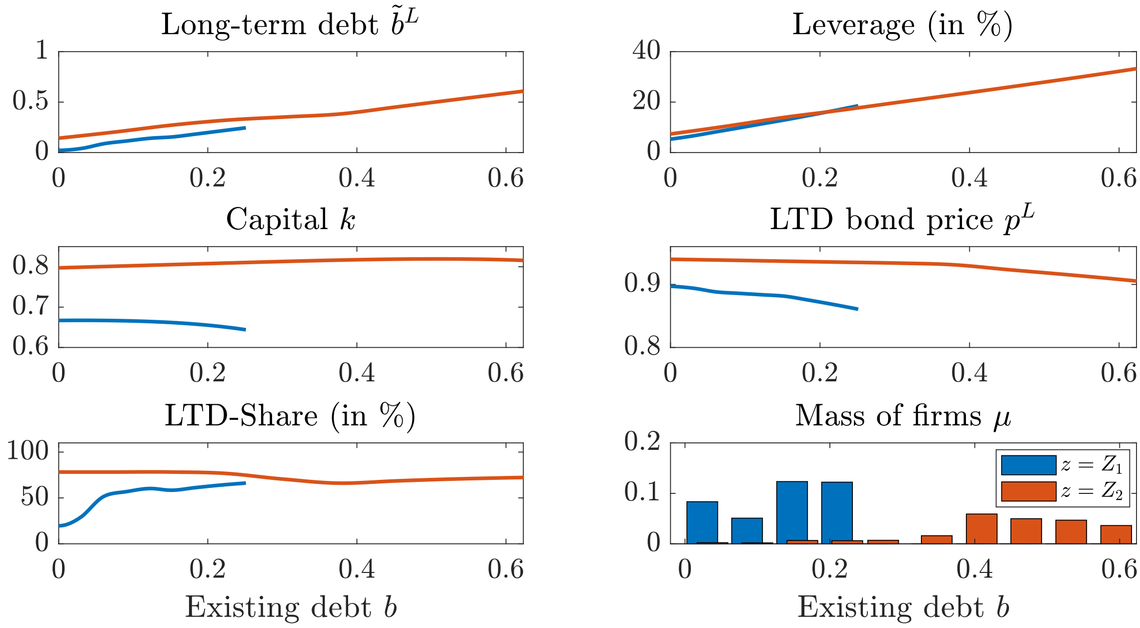


Figure 2: Firm policies

Note: Existing debt b, long-term debt $\tilde{b}^L$, and capital k are normalized by their respective average values in the stationary distribution.

rates. Investment rates have a cross-sectional skewness of 5.2, compared to 4.6 in the data.

3.4. The role of long-term debt at the firm-level

Figure 2 shows the equilibrium policies of firms in the two lowest productivity states Z_1 (blue) and Z_2 (red) as functions of the endogenous state variable b.[10]

Entrants start operating with $z^e = Z_1$ and $b = 0$. Initially, they choose low values of long-term debt $\tilde{b}^L$ and leverage. Subsequent decisions are taken in the presence of existing debt ($b > 0$). This reduces the part of expected default costs which is internalized by firms through the bond market. Firms respond by choosing higher values of $\tilde{b}^L$ and leverage. In this way, firms gradually increase their stock of existing debt over time. This is the *debt dilution* effect described above. At higher values of existing debt b, shareholders capture less of the benefits of investment because of *debt overhang*. As a result, capital falls in b in the lowest productivity state. In response to the associated increase in default risk, the bond price falls in b.

As firms in the lowest productivity state build up long-term debt over time, they also increase their long-term debt share. This is the result of two opposing forces. On the one hand, default risk grows in b, which increases the elasticity of the long-term bond price p^L with respect to changes in future investment and borrowing decisions. This increases the marginal cost of borrowing at long maturities. On the other hand, firms internalize

[10]The calibrated model has 16 productivity levels. Each productivity level has a unique debt grid.

Table 3: Unconditional distributions

	Mean	Percentile		
		25	50	75
Model				
Leverage	32.6	14.6	25.9	46.0
Long-term debt share	66.4	61.4	69.6	76.9
Credit spread on long-term debt	2.4	1.7	2.1	3.3
Data				
Leverage	32.8	3.6	21.9	39.9
Long-term debt share	66.9	44.8	83.1	96.7
Credit spread on long-term debt	2.5	1.1	2.0	3.5

Note: Model moments are computed from the stationary distribution of the model. Source for data moments: Compustat and FISD. See Appendix B for details.

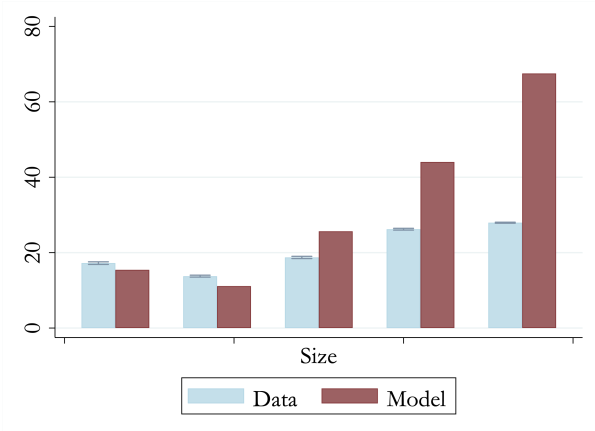
fewer of these costs as b grows because a larger part is borne by existing creditors. This effect encourages firms to choose a higher share of long-term debt as b rises.

Firms in the lowest productivity state build up long-term debt until they reach a stable point. If firms receive a positive productivity shock ($z = Z_2$, red), they increase their scale of production and choose higher values of capital. Profitability increases because the fixed cost of operation is now smaller relative to firm revenues. For given amounts of leverage, default risk is reduced, which translates into a higher market price of firm debt. In addition, the long-term bond price becomes less elastic with respect to changes in future investment and borrowing decisions. This reduces the marginal cost of borrowing at long maturities for high-productivity firms and results in a higher long-term debt share.[11]

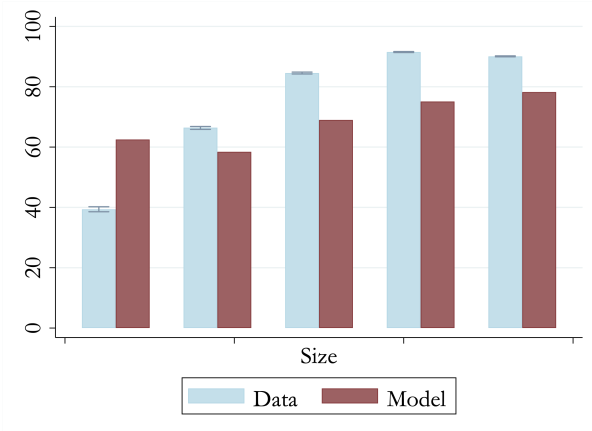
3.5. Long-term debt and the cross-section of firms

Our calibration targeted the average values of leverage, long-term debt shares, and credit spreads. We now show that the model is also consistent with a number of untargeted empirical moments from the conditional and unconditional distributions of leverage, long-term debt shares, credit spreads, firm size, and firm age.

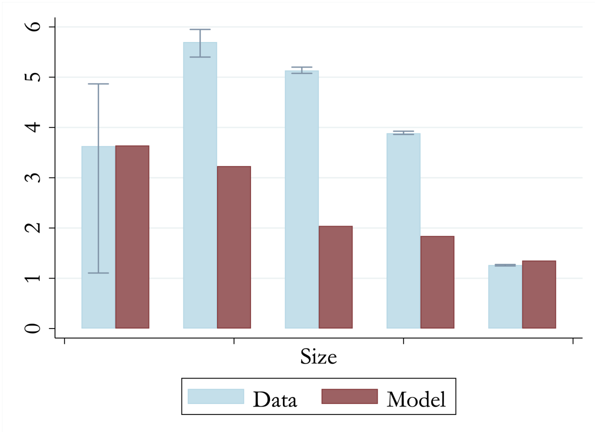
[11] At $z = Z_2$, firms' capital choice is hump-shaped in b. On the one hand, firms choose higher debt levels as b rises which reduces the effective tax burden of the firm and thereby raises the marginal benefit of investment. This effect explains the positive slope of k for low values of b. On the other hand, credit spreads increase and shareholders capture less of the benefits of investment as b rises further (*debt overhang*). This explains why capital begins to fall for higher values of b.



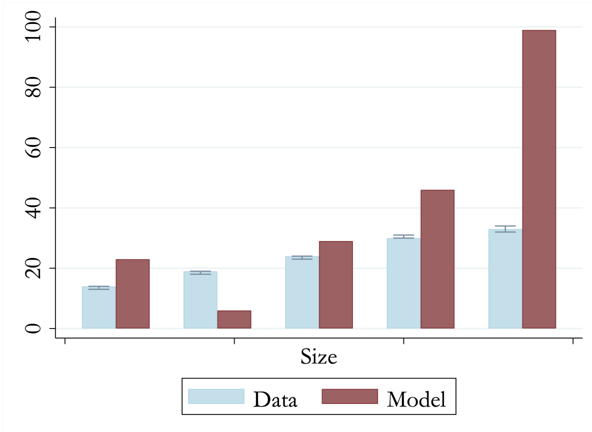
(a) Leverage (in %)



(b) Long-term debt share (in %)



(c) Credit spread on long-term debt (in %)



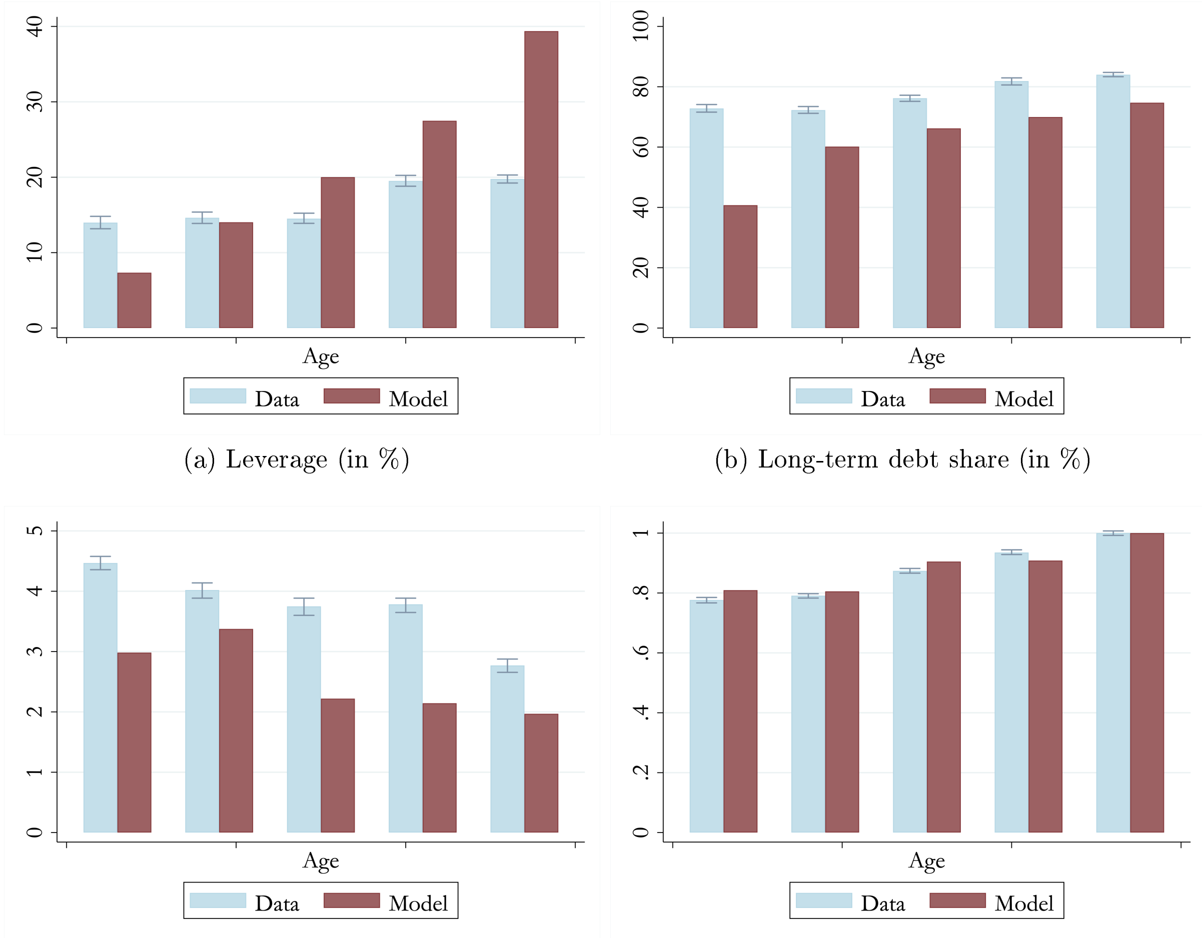
(d) Age (in quarters)

Figure 3: Firm variables conditional on size

Note: For each variable, median values are shown by size quintile. Size is lagged total assets. In the data, age is measured as quarters since IPO date. Data moments are shown together with 95% confidence intervals. See Appendix B for details.

Unconditional distributions Table 3 shows the unconditional distributions of key financial variables in the model and in the data. For leverage, long-term debt shares, and long-term bond spreads, we calculate the mean and the 25th, 50th, and 75th percentiles across firms. While the means were targeted in the calibration, Table 3 shows that the model also produces a significant amount of dispersion across firms. Although the interquartile ranges are somewhat smaller than in the data, the model generates important features of all three distributions. For example, the median values of leverage and credit spreads lie below their respective means while the opposite is true for the long-term debt share.

Size Size is a key dimension of firm heterogeneity. Figure 3 shows how leverage, long-term debt shares, credit spreads, and firm age co-vary with firm size in the data and in



(a) Leverage (in %)

(b) Long-term debt share (in %)

(c) Credit spread on long-term debt (in %)

(d) Size (normalized)

Figure 4: Firm variables conditional on age

Note: For each variable, median values are shown by age quintile. In the data, age is measured as quarters since IPO date. Size is log total firm assets and is normalized to one for the highest age quintile. Data moments are shown together with 95% confidence intervals. See Appendix B for details.

the model. Size is measured as lagged total assets. We group firms into size quintiles and compute median values for each variable by size quintile. The data is shown as the light blue bars. The error bands represent 95% confidence intervals. The red bars show the corresponding values in the model.

Figure 3 shows that larger firms have significantly higher leverage and higher long-term debt shares in the data. While the median share of long-term debt is only about 40% for the lowest size quintile, this value rises to 90% for the largest firms. Larger firms pay lower credit spreads and are older than smaller firms.[12]

Although these moments were not targeted in the calibration, the model replicates

[12]Most small firms in Compustat are unrated which means that we cannot assign a credit spread to them. This explains the large confidence interval for the bottom quintile in panel (c) of Figure 3.

Table 4: Correlation of firm variables with size and age

	Size		Age	
	Data	Model	Data	Model
Leverage	0.71	0.96	0.69	0.97
Long-term debt share	0.86	0.83	0.79	0.79
Credit spread on long-term debt	-0.60	-0.84	-0.80	-0.82
Age	0.94	0.95	-	-
Size	-	-	0.93	0.91

Note: The firm sample is sorted by size (age) and split into 50 equally sized groups. The table reports pairwise correlations between group median values of size (age) and group median values of leverage, the long-term debt share, credit spreads, and age (size). In the data, age is measured as quarters since IPO date. Size is log total firm assets. See Appendix B for details.

these empirical patterns. Larger firms are more profitable in the model because the fixed cost of operation is smaller relative to the scale of production. For given amounts of leverage, higher profitability reduces default risk. This allows larger firms to take on more debt at lower credit spreads. Larger firms also borrow at longer maturities. This is because lower default risk reduces the elasticity of the long-term bond price with respect to changes in future investment and borrowing decisions. The median share of long-term debt rises from about 60% in the lowest size quintile to close to 80% for the highest size group. Finally, larger firms are older because productivity increases with age and default rates are falling in size.

Age A second key dimension of firm heterogeneity is age. In the data, we measure firm age as time passed since a firm's initial public offering (IPO).[13] The blue bars in Figure 4 show that older firms borrow at longer maturities, have higher leverage, and pay lower credit spreads. Firm size increases with age. These patterns are in line with the predictions of the model. As firms grow older they become more productive and increase their scale of production. Their profitability increases, which allows them to borrow more debt at lower credit spreads and longer maturities.

These co-movements are summarized in Table 4. While five bins are used in Figure 3 and 4 to group firms by size and age, in Table 4 the number of bins is 50. The table reports pairwise correlations across firm groups between leverage, the long-term debt share, credit spreads, size, and age. Table 4 confirms the results shown above: size and age are positively related to leverage and the long-term debt share, and negatively related to credit spreads. Our quantitative model replicates these patterns.

[13]Our data set does not contain a firm's actual creation date but it includes the IPO date for many publicly listed firms. This variable allows us to track firm characteristics over time since the initial listing on the stock market.

This section has shown that the model generates a realistic co-movement of debt maturity, leverage, and credit spreads in the cross section of firms. In Appendix C, we show that the model is also consistent with several additional patterns in the composition of firms' external financing flows. For example, the share of long-term debt in net debt issuances increases in firm size both in the data and the model. Small firms rely heavily on equity issuance, whereas large firms are net dividend payers. The overall importance of external financing is declining in size both in the model and the data.

4. Model experiments

The previous section showed that the model predictions are in line with important cross-sectional facts on firm financing and debt maturity. We now use the model to conduct two counterfactual experiments. First, we quantify the costs of debt dilution and debt overhang in distorting firms' investment, borrowing, and maturity choices. In a second experiment, we compare our benchmark model to a standard model in which firms only use short-term debt. We find that accounting for firms' maturity choices can overturn standard results.

4.1. The cost of debt dilution and debt overhang

In the model described in Section 2, the only downside of borrowing long-term is that a higher future outstanding stock of debt distorts future firm behavior because of debt dilution and debt overhang. This cost of long-term debt arises because of a commitment problem. When a firm sells a long-term bond to creditors, it would like to promise to maintain low future levels of debt and high levels of investment in order to increase the revenue raised on the bond market today. But such a promise is not credible. Once a firm has sold its debt and raised the associated revenue, it has no incentive to take the effects of its actions on the market value of existing debt into account. Because creditors have rational expectations, they correctly anticipate and price in the firm's future behavior. This results in a lower price of long-term debt and higher costs of capital for firms.

We now study the effects of this commitment problem. We compare the solution of our benchmark model to a counterfactual economy in which firms internalize the effect of their borrowing and investment decisions on the market value of existing debt. To do so, we modify the firm objective in (12) in the following way:[14]

$$
\begin{aligned}
W(b,z) = & \max_{\phi(b,z)=\left\{\substack{k,l,\tilde{e}\geq\underline{\tilde{e}},\\ \tilde{b}^S,\tilde{b}^L}\right\}} b\,p^L - T(b,z) - \tilde{e} \\
& + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty} \left[q + (1-\kappa)\,\mathbb{E}\,W(b',z') - \kappa\, b'\,\mathbb{E}\,g(b',z')\right]\varphi(\varepsilon)d\varepsilon \qquad (27)
\end{aligned}
$$

[14] See Hatchondo et al. (2016) for a similar exercise in a quantitative model of sovereign default and long-term public debt.

Table 5: The cost of debt dilution and debt overhang

Moment	Benchmark	Counterfactual
Average firm leverage	32.6%	28.0%
Default rate (quarterly)	0.6%	0.4%
Average credit spread on long-term debt	2.4%	1.2%
Average share of long-term debt	66.4%	78.2%
Average capital	1.00	+0.2%
Aggregate capital	1.00	−1.5%
GDP	1.00	+0.9%
Consumption	1.00	+1.5%

Note: Average capital (per firm), aggregate capital, GDP, and consumption are normalized to one in the benchmark economy.

Different from the benchmark firm problem in (12), the market value of existing debt $b\,p^L$ is added to the firm's objective. In order not to mechanically affect the value $W(b,z)$, we introduce a state-contingent lump-sum tax $T(b,z)$ which is specified such that in equilibrium $T(b,z) = b\,p^L$. Firms choose $\phi(b,z) = \{k, l, \tilde{e}, \tilde{b}^S, \tilde{b}^L\}$ to maximize (27) subject to the same set of constraints as in (12).[15]

By adding the market value of existing debt $b\,p^L$ to the firm's objective, we eliminate the commitment problem associated to long-term debt. When the firm chooses capital and debt, it now internalizes the effect of its actions on the market value of existing debt $b\,p^L$. In contrast to Section 2.8, the stock of existing debt b does not enter the first order conditions characterizing a solution to (27) except for the debt issuance cost $H(\tilde{b}^S, \tilde{b}^L, b)$. Apart from debt issuance costs, b has no effect on firm behavior. Neither debt dilution nor debt overhang are present in (27).

Table 5 compares the long-run equilibria of the counterfactual economy and the benchmark economy. When firms internalize the effect of their actions on existing creditors, they choose lower leverage. Because of reduced default risk, they pay lower credit spreads. Credit spreads on long-term debt are half of what they were in the benchmark economy. The maturity structure of firm debt changes as well. The only downside of long-term debt in the benchmark model was that it gave rise to debt dilution and debt overhang. Once this problem is eliminated, firms issue only long-term debt. Given the quarterly repayment rate $\gamma = 0.05$, this corner solution in debt maturity implies that the share of debt with maturity above one year is $(1-\gamma)^4/(1+r)^4 = 78.2\%$.

Lower credit spreads reduce firms' costs of capital and allow them to increase investment and output. The increase in firm capital is dampened by general equilibrium effects, as higher household consumption reduces labor supply and drives up the wage rate.

[15]The value $W(b,z)$ will differ from $V(b,z)$ in (12) only because of different firm behavior. See Appendix D for details.

Table 6: A financial reform

	STD model		LTD model	
Moment	Pre-reform	Post-reform	Pre-reform	Post-reform
Average firm leverage	32.6%	33.3%	32.6%	33.3%
Default rate (quarterly)	0.7%	0.9%	0.6%	0.6%
Average credit spread	2.4%	2.6%	2.5%	2.7%
Average share of LTD	-	-	66.4	68.8%
Average capital	1.00	+0.04%	1.00	+0.10%
Aggregate capital	1.00	+0.09%	1.00	−0.26%
GDP	1.00	+0.02%	1.00	−0.10%
Consumption	1.00	+0.01%	1.00	−0.07%

Note: The table reports the steady state effects of a 25%-reduction of ξ in two model economies: the short-term debt model ('STD model') and the model with endogenous debt maturity ('LTD model'). The average credit spread is the issuance-weighted average of short-term and long-term credit spreads. Average capital (per firm), aggregate capital, GDP, and consumption are normalized to one in the pre-reform equilibria.

4.2. A financial reform

Standard models in macroeconomics assume that all firm debt is short-term. By assumption, debt dilution and debt overhang are absent in that case. This modelling choice can affect model-based policy evaluations. We show this by studying the effects of a financial reform in two different models: (1.) a standard short-term debt model, and (2.) the model of endogenous debt maturity introduced above.

We consider a financial reform which lowers the default cost ξ by 25%. Such a reform could be implemented by speeding up the legal process of bankruptcy, having a more efficient court system, assigning clear control rights to creditors in case of default, etc.

Short-term debt model Consider first a model in which firms only use short-term debt.[16] The columns labeled 'STD model' in Table 6 show the effects of the financial reform on the long-run equilibrium of the economy. Leverage, the default rate, and the average credit spread increase. Capital, GDP, and consumption are higher.

In the short-term debt model, firms roll over the entire stock of debt each period. Through the bond market, they fully internalize all expected costs of default. Leverage is chosen to maximize firm value, that is, the sum of all equity and debt claims. When the default cost ξ is reduced, the trade-off between the tax advantage of debt and expected default costs shifts in favor of higher leverage. After the reform, the default rate is higher but the lower effective tax burden increases investment, output, and consumption.

[16]For the short-term debt model, we set the repayment rate γ to 1. See Appendix D for details.

Endogenous debt maturity The crucial difference in the benchmark economy with long-term debt is that firms do not maximize the sum of all equity and debt claims. Firms internalize the market value of newly issued debt but disregard any effects of their behavior on the market value of existing debt. Because creditors have rational expectations, they pay a low price for long-term debt. This implies high costs of capital for firms. The larger is the equilibrium stock of existing debt, the higher are the costs of debt dilution and debt overhang in the form of elevated credit spreads and depressed investment.

Now consider the financial reform in the benchmark model with long-term debt. The results are shown in the last two columns of Table 6 labeled 'LTD model'. The effects of the reform on aggregate capital, output, and consumption are the opposite of the short-term debt model.

These differences arise because of debt dilution and debt overhang. Just as in the short-term debt model, a reduction in ξ lowers the cost of debt financing and results in higher leverage. In the model with long-term debt, there is an additional effect. As firms borrow more, they increase the level of long-term debt. The steady state level of existing debt is higher, which increases the costs associated to debt dilution and debt overhang. As Table 6 shows, this effect is strong enough for the financial reform to backfire. In the benchmark model with endogenous debt maturity, aggregate investment, output, and consumption are lower after the reform.

5. Conclusion

There is substantial heterogeneity in firms' investment, financing, and maturity choices. These choices shape the firm distribution and determine the effectiveness of economic policy. In this paper, we showed that introducing long-term debt and a maturity choice into a standard model of production, firm financing, and costly default replicates important cross-sectional facts on investment, leverage, credit spreads, and debt maturity. Moreover, this model contains new lessons for policy: A financial reform which increases investment and output in a standard short-term debt model can have the opposite effects in a model of endogenous debt maturity. These results suggest that models of investment and firm financing should take firms' maturity choice into account.

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A. Model appendix

In this section of the appendix, we derive the first order conditions from Section 2.8. We also provide details on the model counterparts of important empirical moments.

A.1. Characterization

In the following, we show that the firm problem (12) can be expressed in terms of three choice variables: the scale of production k, and the amounts of long-term debt $\tilde{b}^L$ and short-term debt $\tilde{b}^S$.

We start by expressing a firm's labor choice as a function of firm capital. For given values of k, $\tilde{e}$, $\tilde{b}^L$, $\tilde{b}^S$, and $\bar{\varepsilon}$, optimal labor demand l^* satisfies:

$$z\,\zeta\,\left(k^{\psi}l^{*1-\psi}\right)^{\zeta-1}(1-\psi)\,k^{\psi}l^{*-\psi}\;-\;w\;=\;0 \tag{A.28}$$

Using (A.28), output net of labor costs $y - wl^*$ can be written as

$$Ak^{\alpha} \equiv z^{\frac{1}{1-\zeta(1-\psi)}}\left(\frac{\zeta(1-\psi)}{w}\right)^{\frac{\zeta(1-\psi)}{1-\zeta(1-\psi)}}\left[1-\zeta(1-\psi)\right]k^{\frac{\psi\zeta}{1-\zeta(1-\psi)}}, \tag{A.29}$$

where:

$$A \equiv z^{\frac{1}{1-\zeta(1-\psi)}}\left(\frac{\zeta(1-\psi)}{w}\right)^{\frac{\zeta(1-\psi)}{1-\zeta(1-\psi)}}\left[1-\zeta(1-\psi)\right], \quad \text{and:} \quad \alpha \equiv \frac{\psi\zeta}{1-\zeta(1-\psi)} \tag{A.30}$$

The next step is to express the threshold value $\bar{\varepsilon}$ in terms of k, $\tilde{b}^L$, and $\tilde{b}^S$. Applying (A.29) to the definition of $\bar{\varepsilon}$ in (7) yields

$$\begin{aligned}(1-\tau)\bar{\varepsilon}k = \tilde{b}^S[1+(1-\tau)c] + \tilde{b}^L[\gamma+(1-\tau)c] - k - (1-\tau)\left[Ak^{\alpha}-\delta k - f\right]\\ -(1-\kappa)\,\mathbb{E}\,V\left(b',z'\right)\;+\;\kappa\,b'\,\mathbb{E}\,g(b',z')\,.\end{aligned} \tag{A.31}$$

Using the expression for capital in (4), we can express $\tilde{e}$ as

$$\tilde{e} = k - p^S\tilde{b}^S - p^L(\tilde{b}^L - b) + H(\tilde{b}^S, \tilde{b}^L, b)\,. \tag{A.32}$$

If long-term debt issuance is positive (i.e. $\tilde{b}^L - b > 0$), debt issuance costs are:

$$H(\tilde{b}^S, \tilde{b}^L, b) = \eta\Big(\tilde{b}^S + \tilde{b}^L - b\Big)^2 \tag{A.33}$$

Assuming $\tilde{b}^L - b > 0$, and applying (A.31) and (A.32) to (12) yields as the firm objective:

$$V(b,z) = -k + p^S \tilde{b}^S + p^L(\tilde{b}^L - b) - \eta(\tilde{b}^S + \tilde{b}^L - b)^2 + \frac{1-\tau}{1+r}\, k \int_{\bar{\varepsilon}}^{\infty} (\varepsilon - \bar{\varepsilon})\varphi(\varepsilon) d\varepsilon \tag{A.34}$$

The short-term bond price depends on $\bar{\varepsilon}$, $\tilde{b}^S$, $\tilde{b}^L$, and k:

$$p^S = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}^L + \tilde{b}^S}\int_{-\infty}^{\bar{\varepsilon}}\Big[k + (1-\tau)\left(Ak^\alpha + \varepsilon k - \delta k - f\right)\Big]\varphi(\varepsilon)d\varepsilon\right] \tag{A.35}$$

Because $\bar{\varepsilon}$ is pinned down through (A.31) by the firm's choice of $\tilde{b}^S$, $\tilde{b}^L$, and k, the short-term bond price likewise only depends on the three choice variables $\tilde{b}^S$, $\tilde{b}^L$, and k. The same reasoning applies to the price of long-term debt:

$$\begin{aligned} p^L = \frac{1}{1+r}\Bigg[&[1-\Phi(\bar{\varepsilon})]\left[\gamma + c + (1-\gamma)\,\mathbb{E}\, g\left(b', z'\right)\right] \\ &+ \frac{(1-\xi)}{\tilde{b}^L + \tilde{b}^S}\int_{-\infty}^{\bar{\varepsilon}}\Big[k + (1-\tau)\left(Ak^\alpha + \varepsilon k - \delta k - f\right)\Big]\varphi(\varepsilon)d\varepsilon\Bigg] \end{aligned} \tag{A.36}$$

It follows that the solution to (12) is found by choosing $\tilde{b}^S$, $\tilde{b}^L$, and k to maximize (A.34) subject to the default threshold in (A.31), and the bond prices in (A.35) and (A.36).

First order conditions

An interior solution to (12) is characterized by three first order conditions. For simplicity, we derive these optimality conditions assuming that long-term debt issuance is positive ($\tilde{b}^L - b > 0$), that there is no exogenous exit ($\kappa = 0$), and that the liquidation value of a firm is zero ($\xi = 1$).

The three choice variables affect the threshold value $\bar{\varepsilon}$ through (A.31). For given values of $\tilde{b}^S$, $\tilde{b}^L$, and k, a marginal increase in $\bar{\varepsilon}$ affects the firm's objective (A.34) according to:

$$\Delta\bar{\varepsilon} \equiv \tilde{b}^S \frac{\partial p^S}{\partial \bar{\varepsilon}} + (\tilde{b}^L - b)\frac{\partial p^L}{\partial \bar{\varepsilon}} - \frac{1-\tau}{1+r}k[1-\Phi(\bar{\varepsilon})] \tag{A.37}$$

$$= -\tilde{b}^S \frac{1+c}{1+r}\varphi(\bar{\varepsilon}) - (\tilde{b}^L - b)\frac{\gamma + c + (1-\gamma)\,\mathbb{E}\, g\left(b', z'\right)}{1+r}\varphi(\bar{\varepsilon}) - \frac{1-\tau}{1+r}k[1-\Phi(\bar{\varepsilon})] \tag{A.38}$$

A higher threshold value $\bar{\varepsilon}$ implies higher default risk. For given values of $\tilde{b}^S$, $\tilde{b}^L$, and k, higher default risk unambiguously reduces shareholder value. First, higher expected default costs reduce the market price of short-term debt ($\partial p^S/\partial\bar{\varepsilon}<0$) and long-term debt ($\partial p^L/\partial\bar{\varepsilon}<0$). This lowers the revenue which the firm raises on the bond market. Second, a higher threshold value $\bar{\varepsilon}$ means that a higher share of firm earnings is paid out to creditors, lowering dividend payments to shareholders.

Capital The firm's first order condition with respect to capital k is:

$$-1+\frac{\partial\bar{\varepsilon}}{\partial k}\,\Delta\bar{\varepsilon}+\frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}(\varepsilon-\bar{\varepsilon})\varphi(\varepsilon)d\varepsilon=0\,, \tag{A.39}$$

$$\text{where:}\qquad \frac{\partial\bar{\varepsilon}}{\partial k} \;=\; -\,\frac{1+(1-\tau)[A\alpha k^{\alpha-1}+\bar{\varepsilon}-\delta]}{(1-\tau)k} \tag{A.40}$$

Short-term debt The firm's first order condition with respect to $\tilde{b}^S$ is:

$$p^S+\frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^S}\,\Delta\bar{\varepsilon}-2\eta(\tilde{b}^S+\tilde{b}^L-b)=\frac{1+c}{1+r}\,[1-\Phi(\bar{\varepsilon})]+\frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^S}\,\Delta\bar{\varepsilon}-2\eta(\tilde{b}^S+\tilde{b}^L-b)=0\,, \tag{A.41}$$

$$\text{where:}\qquad \frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^S} \;=\; \frac{1+(1-\tau)c}{(1-\tau)k}\;>\;0 \tag{A.42}$$

This first order condition can be re-written as:

$$\begin{aligned}&\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,\tau c+\frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^S}\left[\tilde{b}^S\frac{\partial p^S}{\partial\bar{\varepsilon}}+(\tilde{b}^L-b)\frac{\partial p^L}{\partial\bar{\varepsilon}}\right]-2\eta(\tilde{b}^S+\tilde{b}^L-b)=\\&\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,\tau c-\frac{\varphi(\bar{\varepsilon})}{1+r}\,\frac{1+(1-\tau)c}{(1-\tau)k}\left[(1+c)\tilde{b}^S+[\gamma+c+(1-\gamma)\,\mathbb{E}\,g\,(b',z')]\,(\tilde{b}^L-b)\right]\\&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad-2\eta(\tilde{b}^S+\tilde{b}^L-b)=0\end{aligned} \tag{A.43}$$

Long-term debt The firm's first order condition with respect to $\tilde{b}^L$ is:

$$\begin{aligned}&\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,[\gamma+c+(1-\gamma)\,\mathbb{E}\,g\,(b',z')]\\&+(\tilde{b}^L-b)\frac{1-\Phi(\bar{\varepsilon})}{1+r}(1-\gamma)\,\mathbb{E}\,\frac{\partial g(b',z')}{\partial\tilde{b}^L}+\frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^L}\,\Delta\bar{\varepsilon}-2\eta(\tilde{b}^S+\tilde{b}^L-b)=0\,,\end{aligned} \tag{A.44}$$

$$\text{where:}\qquad \frac{\partial\bar{\varepsilon}}{\partial\tilde{b}^L} \;=\; \frac{\gamma+(1-\tau)c-\mathbb{E}\,\frac{\partial V(b',z')}{\partial\tilde{b}^L}}{(1-\tau)k} \tag{A.45}$$

This can be re-written as:

$$\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,\mathbb{E}\Bigg[\tau c+(1-\gamma)\left(g\left(b',z'\right)+(\tilde{b}^L-b)\frac{\partial g\left(b',z'\right)}{\partial \tilde{b}^L}\right)+\frac{\partial V\left(b',z'\right)}{\partial \tilde{b}^L}\Bigg]$$
$$+\frac{\partial \bar{\varepsilon}}{\partial \tilde{b}^L}\left[\tilde{b}^S\frac{\partial p^S}{\partial \bar{\varepsilon}}+(\tilde{b}^L-b)\frac{\partial p^L}{\partial \bar{\varepsilon}}\right]-2\eta(\tilde{b}^S+\tilde{b}^L-b)$$
$$=\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,\mathbb{E}\Bigg[\tau c+(1-\gamma)\left(g\left(b',z'\right)+(\tilde{b}^L-b)\frac{\partial g\left(b',z'\right)}{\partial \tilde{b}^L}\right)+\frac{\partial V\left(b',z'\right)}{\partial \tilde{b}^L}\Bigg]$$
$$-\frac{\varphi(\bar{\varepsilon})}{1+r}\,\frac{\gamma+(1-\tau)c-\mathbb{E}\,\frac{\partial V(b',z')}{\partial \tilde{b}^L}}{(1-\tau)k}\,\Big[(1+c)\tilde{b}^S+\left[\gamma+c+(1-\gamma)\,\mathbb{E}\,g\left(b',z'\right)\right](\tilde{b}^L-b)\Big]$$
$$-2\eta(\tilde{b}^S+\tilde{b}^L-b)\;=\;0 \quad \text{(A.46)}$$

Combining the two first order conditions for short-term debt (A.43) and long-term debt (A.46) yields a condition for firms' optimal maturity choice:

$$\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,\mathbb{E}\Bigg[(1-\gamma)\left(g\left(b',z'\right)+(\tilde{b}^L-b)\frac{\partial g\left(b',z'\right)}{\partial \tilde{b}^L}\right)+\frac{\partial V\left(b',z'\right)}{\partial \tilde{b}^L}\Bigg]$$
$$-\frac{1-\gamma+\mathbb{E}\,\frac{\partial V(b',z')}{\partial \tilde{b}^L}}{(1-\tau)k}\left[\tilde{b}^S\frac{\partial p^S}{\partial \bar{\varepsilon}}+(\tilde{b}^L-b)\frac{\partial p^L}{\partial \bar{\varepsilon}}\right]=0 \qquad \text{(A.47)}$$

An increase in the quantity of long-term bonds $\tilde{b}^L$ implies a higher stock of existing debt tomorrow: $b'=(1-\gamma)\tilde{b}^L$. Using (A.34), we derive:

$$\frac{\partial V(b,z)}{\partial b}=2\eta(\tilde{b}^S+\tilde{b}^L-b)-p^L \qquad \text{(A.48)}$$

It follows that:

$$\frac{\partial V(b',z')}{\partial \tilde{b}^L}=(1-\gamma)\left[2\eta(\tilde{b}^{S'}+\tilde{b}^{L'}-b')-g\left(b',z'\right)\right] \qquad \text{(A.49)}$$

Using (A.49), the optimality condition for firms' maturity choice (A.47) becomes:

$$\frac{1-\Phi(\bar{\varepsilon})}{1+r}\,\mathbb{E}\left[(1-\gamma)2\eta(\tilde{b}^{S'}+\tilde{b}^{L'}-b')+(\tilde{b}^L-b)(1-\gamma)\frac{\partial g\left(b',z'\right)}{\partial \tilde{b}^L}\right]$$
$$-\frac{1-\gamma}{(1-\tau)k}\,\mathbb{E}\left[1-g(b',z')+2\eta(\tilde{b}^{S'}+\tilde{b}^{L'}-b')\right]\left[\tilde{b}^S\frac{\partial p^S}{\partial \bar{\varepsilon}}+(\tilde{b}^L-b)\frac{\partial p^L}{\partial \bar{\varepsilon}}\right]=0 \quad \text{(A.50)}$$

A.2. Model moments

The total amount of firm debt D is the present value of future debt payments discounted at the quarterly riskless rate r:

$$\begin{aligned} D &\equiv \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L + (1-\gamma)\frac{\gamma+c}{(1+r)^2}\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\tilde{b}^L + ... \\ &= \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L\sum_{j=0}^{\infty}\left(\frac{1-\gamma}{1+r}\right)^j = \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\tilde{b}^L \end{aligned} \tag{A.51}$$

The long-term debt share of a given firm is the present value of debt payments due more than four quarters from today divided by the total amount of firm debt D:

$$\begin{aligned} &\frac{1}{D}\left((1-\gamma)^4\frac{\gamma+c}{(1+r)^5}\tilde{b}^L + (1-\gamma)^5\frac{\gamma+c}{(1+r)^6}\tilde{b}^L + ...\right) \\ =&\frac{1}{D}\frac{\gamma+c}{\gamma+r}\left(\frac{1-\gamma}{1+r}\right)^4\tilde{b}^L \end{aligned} \tag{A.52}$$

Because $c = r$ in the stationary equilibrium of our calibrated economy, the long-term debt share simplifies to:

$$\left(\frac{1-\gamma}{1+r}\right)^4\frac{\tilde{b}^L}{\tilde{b}^S + \tilde{b}^L} \tag{A.53}$$

The Macaulay duration is the weighted average term to maturity of the cash flows from a bond divided by the price:

$$\mu = \frac{1}{p_r^L}\sum_{j=1}^{\infty} j(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{p_r^L}\frac{1+r}{(\gamma+r)^2} \tag{A.54}$$

where p_r^L is the price of a riskless long-term bond:

$$p_r^L = \sum_{j=1}^{\infty}(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{r+\gamma} \tag{A.55}$$

It follows for the Macaulay duration:

$$\mu = \frac{1+r}{\gamma+r} \tag{A.56}$$

The short-term spread compares the annual gross return (in the absence of default) from buying a short-term bond with the annualized quarterly riskless rate:

$$\left(\frac{1+c}{p^S}\right)^4 - (1+r)^4 \tag{A.57}$$

The long-term spread compares the annual gross return (in the absence of default and assuming p^L is constant) from buying a long-term bond with the annualized quarterly riskless rate:

$$\left(\frac{\gamma + c + (1-\gamma)p^L}{p^L}\right)^4 - (1+r)^4 = \left(\frac{\gamma + c}{p^L} + 1 - \gamma\right)^4 - (1+r)^4 \tag{A.58}$$

The investment rate is defined as capital expenditures divided by lagged firm capital: $(k_{t+1} - (1-\delta)k_t)/k_t$.

B. Data appendix

In this section, we describe the data set used in Section 3. We construct our firm sample by merging the annual and the quarterly Compustat database for the years 1984-2018. We delete firms that are not incorporated in the US, firms in the financial and public sectors, as well as utilities (SIC codes 6000-6999, 9000-9999, and 4900-4949). Where appropriate, firm-level observations have been deflated using the CPI.[17]

Our cleaning procedure largely follows Covas and Den Haan (2011). We drop observations with negative total assets (*atq*) or negative sales (*saleq*). Observations with negative short-term debt (*dlcq*) or negative long-term debt (*dlttq*) are set to missing. We remove four large firms that were heavily affected by a 1988 accounting change (General Electric (*gvkey* = 005047), Ford (*gvkey* = 004839), Chrysler (*gvkey* = 003022), and General Motors (*gvkey* = 005073)). Firms with more than 50% sales growth between one quarter and the next due to a merger are dropped from the sample (*saleq_fn1*), as are firms that violate the accounting identity (assets = equity + liabilities) by more than 10% of the book value of assets. We delete firms with gaps in the sample and firms with less than five quarters of data.

Balance sheet variables We create the following new variables in the database. Total debt is computed as the sum of debt in current liabilities (*dlcq*) and long-term debt (*dlttq*). The long-term debt share is the ratio of long-term debt to total debt. Book leverage is the ratio of total debt to total assets (*atq*). Observations with leverage ratios of more than 20 are set to missing. Investment is defined as capital expenditures (*capxy*) minus sales of property (*sppey*). The two components of investment are defined as year-to-end variables (cumulative sums) in Compustat and are converted into

[17] We use the consumer price index for all urban consumers available at `https://fred.stlouisfed.org/series/CPIAUCSL`.

quarterly frequency (*capxq* and *sppeq*). A firm's investment rate is constructed by dividing investment by lagged property, plant, and equipment, i.e. (*capxq* - *sppeq*)/l.*ppegtq*. Observations with investment rates in excess of one or below negative one are set to missing.

Our final sample consists of 324,825 observations of 7,859 unique firms.

Credit spreads For data on corporate bond spreads, we use the Mergent Fixed Income Securities Database (FISD). This collection of datasets contains information on corporate bond issues (e.g. yield to maturity, credit rating, maturity date). We select bonds that were issued in US dollars by US firms with the same restrictions of SIC-codes and years which we used for the Compustat sample. Bond issues with missing offering or maturity dates are deleted. We select senior bonds that are classified as US corporate debentures or medium-term notes. The FISD database contains ratings by all major rating agencies. To be consistent with the information in Compustat, we assign to each rating the corresponding Standard & Poor's rating (see Johnson, 2003).

We calculate bond spreads as bond yield at issuance minus the yield of a US treasury of identical maturity issued on the same day. Corporate bonds whose maturity falls in between maturities available for US treasury bonds are assigned a weighted treasury yield.[18] We delete spreads that are below 5 basis points or above 3,500 basis points.

We use this information to construct a quarterly panel of corporate bond spreads by credit rating. For each rating category and each quarter, we compute the median bond spread across all bond issues. These time series of corporate bond spreads broken down by rating class are based on 31,063 bond-level observations. We use this data to proxy a firm's credit spread in a given quarter by the median spread of the corresponding rating class. Quarterly firm-level Standard & Poor's credit ratings are obtained from the Compustat Monthly Updates. Because the FISD includes only bonds with maturity above one quarter, this data is informative with respect to long-term credit spreads in our model.

Tables 2 and 3 To compute the data moments in Tables 2 and 3 we proceed as follows. Every quarter, we compute the mean, p25, p50, and p75 of the distributions of leverage, the long-term debt share, and the credit spread on long-term debt. Then we report the median of each time series. For the investment rate moments in Table 2, we compute each firm's mean investment rate over time as well as its standard deviation and then report the median across all firms. To ensure consistency of the data with our stationary model, we compute firm-level investment rates net of the aggregate investment rate, that is, we subtract the aggregate investment rate of a given year from firm-level investment rates. The skewness of investment rates reported in footnote 6 on page 15 and on page 17 is computed from the cross-section of investment rates in the model and in the data.

[18]Data on US Treasury bonds is available in series H-15 - Selected interest rates at `https://www.federalreserve.gov/datadownload/`.

Figures 3 - 4 The data shown in Figures 3 and 4 was computed as follows. In each quarter, we divide firms into quintiles based on size or age. We then calculate quintile-specific median values of firm-level variables. The figures include 95% confidence intervals. In Figure 3, firm size is lagged total assets ($l.atq$). Firm age is time passed since a firm's IPO date (*ipodate*). In Figure 4, we remove firm observations prior to the IPO date. Firm size is log total assets and is normalized to one for the highest age quintile. Because the number of observations in falling in age, we exclude information beyond the age of sixty quarters.

Table 4 The empirical correlations shown in Table 4 were computed as follows. Each quarter, we sort firms into 50 size and age bins of ascending order, each containing an equal number of firms. For each size and age bin, we calculate group-specific median values of firm-level variables. Table 4 reports correlations of these median values across size and age bins. Firm size is log total assets (lagged by one quarter in the first column of Table 4). Firm age is time passed since a firm's IPO date. In the third column of Table 4, we remove firm observations prior to the IPO date and exclude information beyond the age of sixty quarters.

Model moments The model moments used throughout Section 3 were computed from the stationary distribution and a simulated panel of firms. The simulation is used to compute firm-level investment rates and the model results reported in Figure 3, Figure 4, and Table 4. The simulated panel consists of about 500,000 individual firms. The median firm in the simulated panel operates for 29 quarters. We treat the simulated data in the same way as the empirical firm sample, e.g. we remove firms that exit before the age of five quarters and we exclude information beyond the age of sixty quarters in Figure 4 and in the fourth column of Table 4. Size is log firm capital k and age is time passed since entry.

C. Additional quantitative results

In this section, we present additional empirical and quantitative results on the composition of firms' external financing flows. Figure 5 shows the relationship between firm size and four measures of financing flows: the investment rate, the long-term debt share of debt issuance, the equity share of external financing, and external finance as a fraction of total firm assets. Figure 6 reports the corresponding results broken down by firm age.

As in Figures 3 and 4, we divide firms into quintiles based on size and age. For the investment rate, we calculate quintile-specific median values. For the remaining three variables, we follow a different approach. Firm-level measures of these flow variables display large idiosyncratic variation which may cloud systematic correlations with other variables. Instead of studying quintile-specific median values, we reduce idiosyncratic noise by aggregating flow variables at the quintile-level (see Covas and Den Haan, 2011). Figures 5 and 6 report the median of each of these quintile-specific time series. The figures include 95% confidence intervals.

Figure 5 shows that the model is consistent with several patterns in the composition of firms' external financing flows. For example, the share of long-term debt in net debt issuances increases in firm size both in the data and the model. Small firms rely heavily on equity issuance, whereas large firms are net dividend payers. Figure 6 documents a similar pattern for firm age. The overall importance of external financing flows is declining in size and in age both in the model and the data.

D. Model experiments

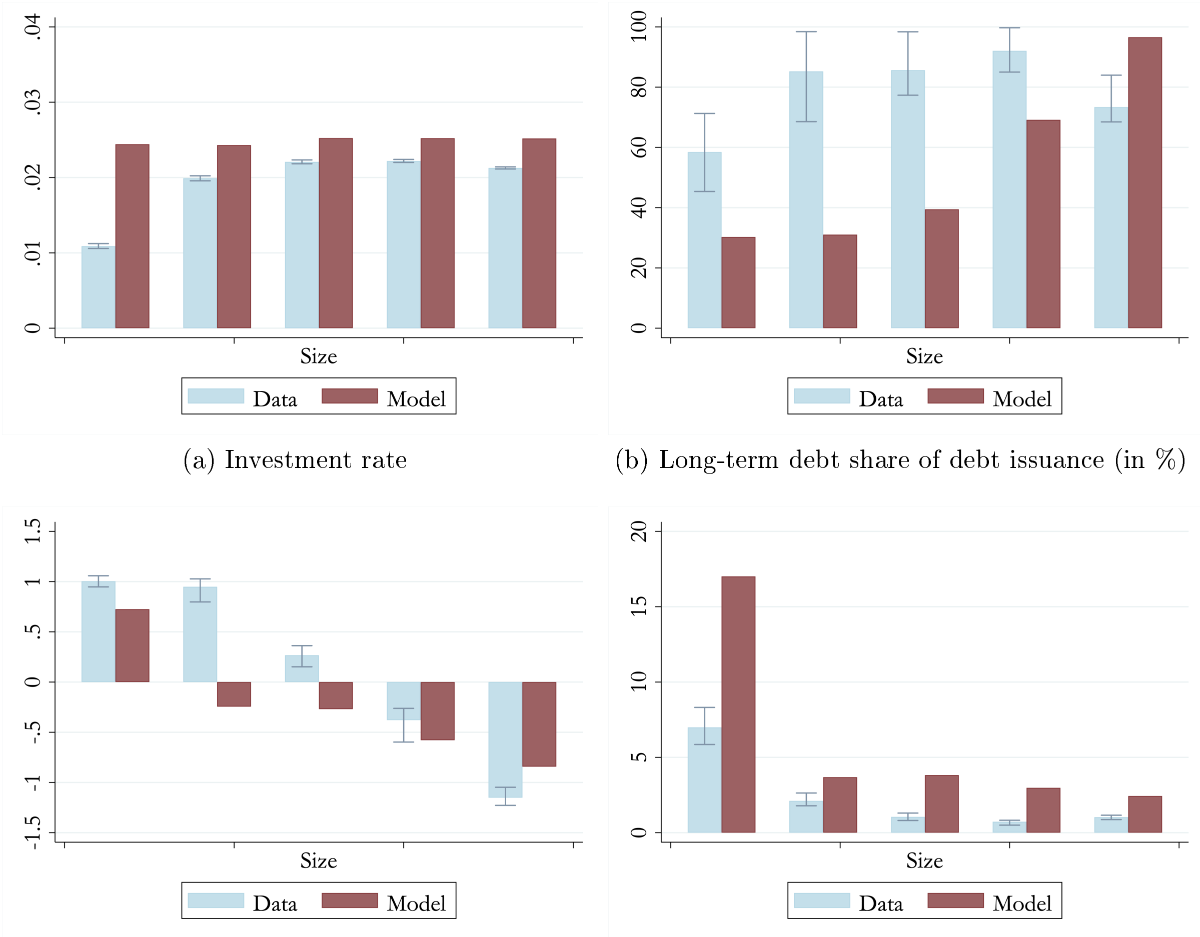
In this section, we lay out the details of the counterfactual firm problem used in Section 4.1 and the short-term debt model of Section 4.2.

D.1. The cost of debt dilution and debt overhang

The only difference between the stationary equilibrium defined in Section 2 and the counterfactual economy studied in Section 4.1 lies in the nature of the firm problem. The value function $V(b,z)$ in (12) is replaced by the value $W(b,z)$ which solves:

$$\begin{aligned} W(b,z) = & \max_{\phi(b,z)=\left\{ \substack{k,l,\tilde{e}\geq\underline{\tilde{e}},\\ \tilde{b}^S,\tilde{b}^L} \right\}} b\,p^L - T(b,z) - \tilde{e} \\ & \quad + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty} \left[q + (1-\kappa)\,\mathbb{E}\,W(b',z') - \kappa\, b'\,\mathbb{E}\,g(b',z')\right]\varphi(\varepsilon)d\varepsilon \qquad \text{(A.59)} \\ \text{s.t.:} \quad & q = k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)\left[y + \varepsilon k - wl - \delta k - f - c(\tilde{b}^S + \tilde{b}^L)\right] \\ & y = z\left(k^{\psi} l^{1-\psi}\right)^{\zeta} \\ & \bar{\varepsilon}: \quad q + (1-\kappa)\,\mathbb{E}\,W(b',z') - \kappa\, b'\,\mathbb{E}\,g(b',z') = 0 \\ & k = \tilde{e} + \tilde{b}^S p^S + (\tilde{b}^L - b)p^L - H(\tilde{b}^S, \tilde{b}^L, b) \\ & b' = (1-\gamma)\tilde{b}^L \\ & p^S = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\;\varphi(\varepsilon)d\varepsilon\right] \\ & p^L = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})]\left[\gamma + c + (1-\gamma)\mathbb{E}\,g(b',z')\right] + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}} \underline{q}\;\varphi(\varepsilon)d\varepsilon\right] \end{aligned}$$

The state-contingent lump-sum tax $T(b,z)$ in (A.59) is specified such that in equilibrium: $T(b,z) = p^L b$. This makes sure that $W(b,z)$ differs from the value $V(b,z)$ in the decentralized model only because of different firm behavior.



(a) Investment rate

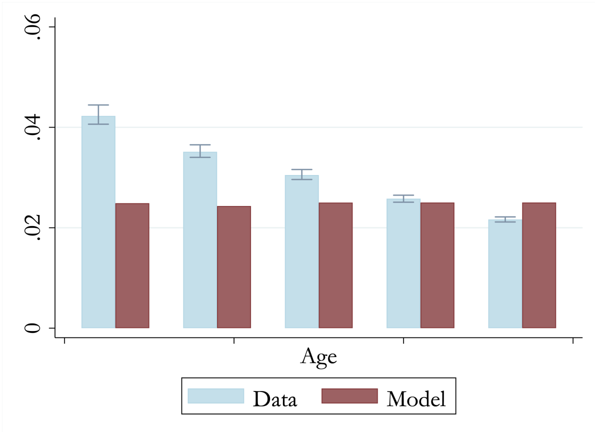
(b) Long-term debt share of debt issuance (in %)

(c) Equity share of external financing

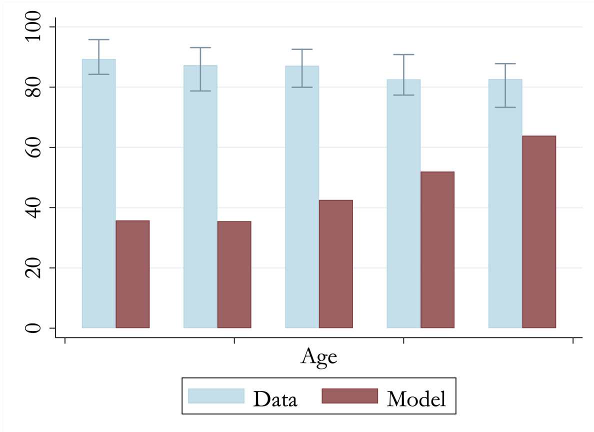
(d) External finance as fraction of total assets (in %)

Figure 5: Flow variables conditional on size

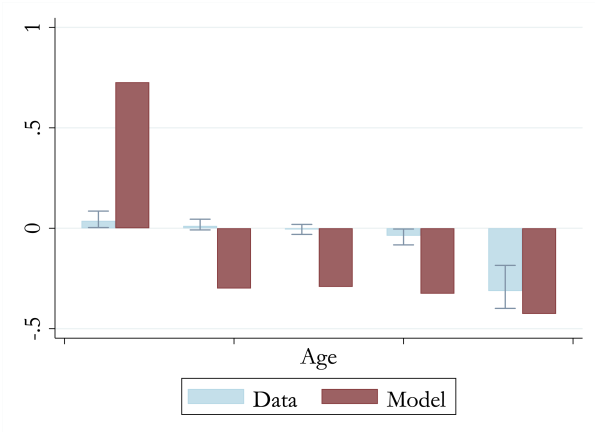
Note: Size is lagged total assets. The investment rate is the median value by size quintile. The remaining three variables are calculated using aggregate data at the quintile-level. For each of these quintile-specific time series, median values are shown by size quintile. Data moments are shown together with 95% confidence intervals. *Data definitions:* The investment rate is defined as capital expenditures (net of sales of property) divided by lagged property, plant, and equipment. The long-term debt share of debt issuance is calculated as the increase in long-term debt divided by the increase in total debt. The equity share of external financing is net equity issuance divided by the absolute value of the sum of net equity issuance and the increase in total debt. Net equity issuance is defined as in Begenau and Salomao (2018): equity issuances (*sstkq*) minus dividend payout (*dvy*) and repurchases (*prstkcq*). External finance as a fraction of total assets is calculated as the absolute value of the sum of net equity issuance and the increase in total debt divided by lagged total firm assets.



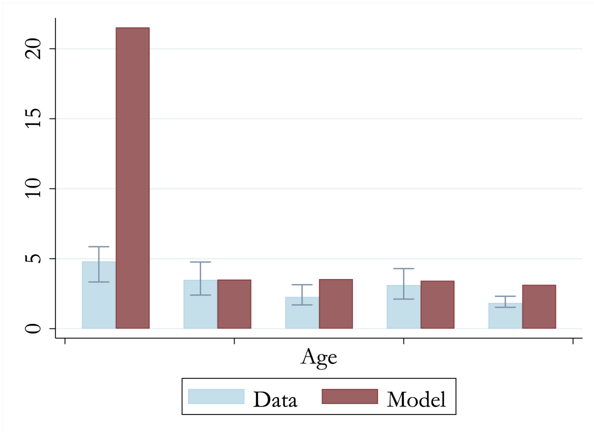
(a) Investment rates



(b) Long-term debt share of debt issuance (in %)



(c) Equity share of external financing



(d) External finance as fraction of total assets (in %)

Figure 6: Flow variables conditional on age

Note: In the data, age is measured as quarters since IPO date. The investment rate is the median value by age quintile. The remaining three variables are calculated using aggregate data at the quintile-level. For each of these quintile-specific time series, median values are shown by age quintile. Data moments are shown together with 95% confidence intervals. *Data definitions:* The investment rate is defined as capital expenditures (net of sales of property) divided by lagged property, plant, and equipment. The long-term debt share of debt issuance is calculated as the increase in long-term debt divided by the increase in total debt. The equity share of external financing is net equity issuance divided by the absolute value of the sum of net equity issuance and the increase in total debt. Net equity issuance is defined as in Begenau and Salomao (2018): equity issuances (*sstkq*) minus dividend payout (*dvy*) and repurchases (*prstkcq*). External finance as a fraction of total assets is calculated as the absolute value of the sum of net equity issuance and the increase in total debt divided by lagged total firm assets.

D.2. Short-term debt model

Except for parameter values, the setup of the short-term debt model used in Section 4.2 is identical to the benchmark model. The key difference is that firms cannot issue long-term debt now: $\gamma = 1$. We adjust the values of σ_ε and ξ in order to match the same average leverage ratio and credit spread as in the benchmark model. Because we no longer target the long-term debt share, we set the issuance cost parameter $\eta = 0$. We change the value of the fixed cost of operation f to generate a unit mass of firms in equilibrium. Table 7 summarizes all parameter changes with respect to Table 2.

Table 7: Short-term debt model - Parameter changes

Parameter	Value	Target	Data	Model
γ	1	-	-	-
σ_ε	0.620	Average firm leverage	32.8%	32.6%
ξ	0.095	Average credit spread	2.5%	2.4%
η	0	-	-	-
f	0.300	Unit mass of firms	-	1.00

Note: Leverage is from Compustat. Credit spreads are computed using data from Compustat and FISD.